Final meeting of the ANR project PERISTOCH

Orléans, 28–29 November 2024

IDP, Bâtiment de Mathématiques, Salle de Séminaires

Program

Thursday 28 November

9h30 – 10h00 : Welcome and coffee

10h00 – 10h50 : Eric Luçon (IDP Orláns), Christophe Poquet (ICJ Lyon)
Long term stability of traveling waves for interacting Hawkes processes on the real line

11h00 – 11h50 : Bastien Fernandez (LPSM Paris)
Feedback-delay dependence of the stability of cluster periodic orbits in populations of degrade-and-fire oscillators with common activator

12h00 – 14h00 : Lunch break

14h00 – 14h55 : Lluís Hernández Navarro (Leeds)
Eco-evolutionary dynamics of cooperative antimicrobial resistance in time-varying environments with spatial structure

15h00 – 15h30 : Coffee break

15h30 – 16h10 : Ali Ellouze (LPSM Paris)
Population dynamics in fresh product markets : Modelling and mathematical analysis

16h20 – 17h10 : Samuel Herrmann (LMB Dijon)
Approximation of first exit times for linear diffusion processes

Friday 29 November

9h30 – 10h00 : Welcome and coffee

10h00 – 10h50 : Noé Cuneo (LPSM Paris)
Large deviations of return times and related entropy estimators on shift spaces

11h00 – 11h40 : Léo Daures (LPSM Paris)
A large deviation principle for discrete, possibly reducible, Markov chains

11h50 – 13h50 : Lunch break

13h50 – 14h40 : Baptiste Bergeot (INSA Blois)
Effects of noise on the dynamics of two types of fast-slow mechanical systems

14h50 – 15h40 : Maxime Mikikian (GREMI Orléans), Julien Barré/Nils Berglund (IDP Orléans), Hiroshi Horii (LJLL Paris)
Modeling mixed-mode oscillations in dusty plasmas: progress report

15h50 – 16h30 : Coffee break

Abstracts

Julien Barré, Nils Berglund, Hiroshi Horii, Maxime Mikikian : *Modeling mixed-mode oscillations in dusty plasmas: progress report*
Mixed-mode oscillations are not necessarily periodic oscillation patterns, composed of both small-amplitude and large-amplitude oscillations. The appearance of these patterns is well understood in some particular systems, such as the FitzHugh–Nagumo model for neuron dynamics, both without and with noise. Very similar oscillation patterns have been experimentally observed in so-called dusty plasmas. In that case, however, we do not yet have a simple mathematical model explaining the observations. We will report on ongoing work, aiming at developing such a model. Starting from fluid equations for the plasma, we arrive, after simplifications, at a drift-diffusion partial differential equation, coupled to a Poisson equation for the electric field. We will show evidence, both analytical and numerical, for the existence of attracting periodic solutions in some cases. These solutions alternate between phases in which the dust density approaches a shock, and rarefaction phases in which the density approaches a more homogeneous profile.

Baptiste Bergeot : *Effects of noise on the dynamics of two types of fast-slow mechanical systems*
This presentation will explore the impact of noise on the dynamics of two types of fast-slow mechanical systems. The first system is a nonlinear passive vibration absorber, referred to as a Nonlinear Energy Sink (NES), which is coupled to a self-sustained oscillator requiring vibration attenuation. The second system is a self-sustained wind musical instrument (such as the clarinet) in which one of the bifurcation parameters – specifically, the air pressure within the mouthpiece of the instrument, due to the musician's breath – varies over time accounting for attack transients performed by the musician. Although these two systems differ greatly in term of applications, they are both modeled by systems of differential equations featuring a small parameter highlighting their singularly perturbed, fast-slow nature. As a result, their dynamics (i) cannot be fully explained by the concepts traditionally used in engineering such as bifurcation diagrams and basins of attraction and (ii) can be significantly influenced by the presence of noise. For each of these systems, the deterministic dynamics will first be described. Subsequently, the influence of noise will be analyzed using both numerical simulations and analytical methods.

Noé Cuneo : *Large deviations of return times and related entropy estimators on shift spaces*

The study of return times in dynamical systems has a long history, and their role as entropy estimators is well established. While considerable attention has been devoted to the corresponding Law of Large Numbers, Central Limit Theorem and pressure function in the literature, surprisingly little was known about their large deviations. In fact, only local versions of the Large Deviation Principle (LDP) were obtained, under some rather restrictive assumptions. I will talk about a recent work in collaboration with Renaud Raquépas (`arXiv:2306.05277`, Commun. Math. Phys. 405(6)), where we prove that return times satisfy the full LDP for a large class of invariant measures on shift spaces. Our assumptions cover in particular irreducible Markov chains, equilibrium measures for Bowen-regular potentials, invariant Gibbs states for absolutely summable interactions in statistical mechanics (including in the phase-transition regime), and some repeated quantum measurement processes. As we will see in a striking example, the rate function governing the LDP can be nonconvex.

Léo Daures : *A large deviation principle for discrete, possibly reducible, Markov chains*
This talk is part of my ongoing PhD research. I will begin by presenting the subadditive method, an elegant approach for deriving large deviation principles (LDPs). This method is particularly effective for systems with strong decoupling assumptions, such as irreducible Markov chains, where it has become standard under strong recurrence conditions. However, when these assumptions are relaxed – specifically when the Markov chain is no longer irreducible – further adaptations of the method are required. I will highlight the additional challenges posed by this setting and discuss a construction that addresses them, enabling the use of the subadditive method. This construction ultimately leads to the derivation of a weak LDP for any Markov chain on a discrete state space.

Ali Ellouze : *Population dynamics in fresh product markets : Modelling and mathematical analysis*
We introduce and study a class of discrete time models for the buyers' population dynamics in a market with N competing sellers. Buyers are partly loyal to previously visited sellers and partly sensitive to sellers' attractiveness, which is a function of price. The sellers in turn adapt their prices depending on the relative volumes of clientele, introducing negative feedback in the dynamics. We provide, a sufficient condition on the system characteristics, for the local asymptotic stability alongside with a counter example of instability when this condition is violated. We also prove that global stability must occur under a concavity type condition, as long as seller attractiveness is not indifferent. Finally, if the time permits, we introduce a second model that could be seen as an improvement of the former and give a flavor of what type of (different) dynamics it can have.

Bastien Fernandez : *Feedback-delay dependence of the stability of cluster periodic orbits in populations of degrade-and-fire oscillators with common activator*
Feedback delay plays a crucial role in the quorum sensing synchronization of synthetic gene oscillators.While this role has been evidenced at the theoretical level in a simplified system of degrade-and-fire oscillators coupled via a common activator protein, full mathematical certifications remained to be provided. In this talk, I will prove

for the very same model that the synchronized degrade-and-fire oscillations are, in absence of delay, unstable in every arbitrarily large population, and are otherwise asymptotically stable for every positive delay. The proof is part of an extensive study of the population dynamics, which puts emphasis on analyzing the dependence on the number of oscillators. Joint work with Matteo Tanzi.

Lluís Hernández Navarro : *Eco-evolutionary dynamics of cooperative antimicrobial resistance in time-varying environments with spatial structure*
Antimicrobial resistance to drugs (AMR), a global threat to human and animal health, is often regarded as a result of microbial cooperation, leading to the coexistence of drug-resistant and drug-sensitive cells in large communities and static environments. This picture is, however, greatly altered by the fluctuations arising in volatile environments, in which microbial communities commonly evolve. Motivated by the need to better understand the "eco-evolution" of cooperative AMR, we study a population of time-varying size consisting of two competing strains: resistant versus sensitive microbes. We model the time-fluctuating environment as random switches between states of nutrient abundance and scarcity, which can cause population bottlenecks. The eco-evolutionary dynamics of cooperative AMR is thus characterised by demographic noise (microbial birth and death events) coupled to environmental fluctuations. In the first part of the talk, I will focus on well-mixed microbial populations. Using computational and analytical means, I will discuss the environmental conditions for the long-lived coexistence or fixation of both strains, and characterise a fluctuation-driven AMR eradication mechanism where resistant microbes experience bottlenecks leading to extinction. In the second part of the talk, I will extend the above results to spatially structured microbial metapopulations, modelled by a 2D square lattice with migration across adjacent populations.

Samuel Herrmann : *Approximation of first exit times for linear diffusion processes*
In this talk a new technique for the path approximation of stochastic processes is presented. The results apply to the Brownian motion and to some families of stochastic differential equations whose distributions could be represented as a function of a time-changed Brownian motion (usually known as L and G-classes). I propose an explicit and easy-to-implement procedure that jointly constructs the sequences of exit times and corresponding exit positions of some well-chosen domains (the random walk on heat balls). This procedure makes it possible to approximate the first moment when Brownian motion exits a given domain, and thus to numerically solve the initial boundary value problem for the heat equation. It also makes it possible to construct a strong ε-approximation of specific one-dimensional diffusion processes. This talk is based on joint works with M. Deaconu and N. Massin.

Eric Luçon, Christophe Poquet : *Long term stability of traveling waves for interacting Hawkes processes on the real line*
We consider a system of interacting Hawkes processes with spatial interaction on the real line, modelling the spiking activity of neurons. On a bounded time interval, the membrane potential profile of the system is shown to converge to the solution of the Neural Field Equation (NFE). Previous results of Ermentrout et al. and Stannat et al.

have shown the existence of traveling wave solution to the NFE that are locally stable. We take advantage of this local stability to prove long term proximity of the microscopic particle system wrt the manifold of traveling waves solutions, together with a characterisation of the random dephasing induced by noise on a long time scale. We borrow here previous arguments used in a context of interacting diffusions and Normally Hyperbolic Manifolds, the main difficulty being to adapt them in a context of interacting point processes.