On simple dual braids and Mikado braids of type D_n

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Coxeter groups and their Artin-Tits groups

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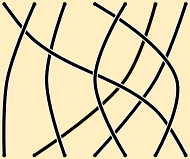

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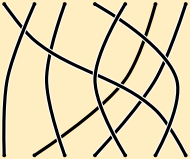

- For $w = s_1 s_2 \cdots s_k \in W$ with $\ell(w) = k$, the lift $\mathbf{s_1 s_2 \cdots s_k}$ in $B(W)$ is well-defined and denoted by $\mathbf{w}$.

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- Mikado braids have nice categorifications in Rouquier's 2-braid group. The images of these braids in the Hecke algebra of the Coxeter system satisfy nice positivity properties.

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- One can show that $B_c^* \hookrightarrow \mathrm{Frac}(B_c^*) \cong B(W)$ and that B_c^* is a Garside monoid. The image of $i_c(s)$ in $B(W)$ is $\mathbf{s}$ for all $s \in S$. It is difficult in general to have formulas for elements $i_c(t)$, $t \in T \backslash \{S\}$ in the classical Artin generators.

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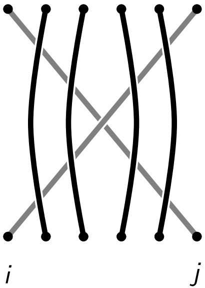

FIGURE 1: The Birman-Ko-Lee generator corresponding to the transposition (i,j).

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- We call the elements of $\mathbf{D}_c$ the *simple dual braids*.

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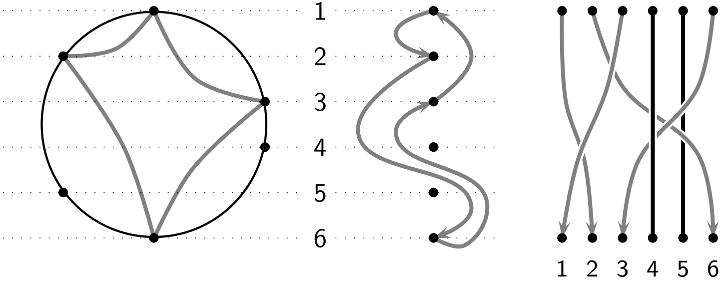

FIGURE 2: A noncrossing partition of $c = (1\ 3\ 4\ 6\ 5\ 2)$ and the corresponding simple dual braid viewed in $B(W)$.

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Theorem (Digne-G., 2017)

Let (W, S) be a finite irreducible Coxeter system of type different from D_n and $c \in W$ a Coxeter element. Let $x \in \mathbf{D}_c$. Then x is a Mikado braid.

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In particular, every simple dual braid in every Artin-Tits group of spherical type is a Mikado braid.

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- Let $n \geq 3$. The Coxeter group (W_{D_n}, S_{D_n}) of type D_n can be realized as an index two subgroup of the Coxeter group (W_{B_n}, S_{B_n}) of type B_n. This embedding is not induced by an embedding (satisfying some expected properties) of the corresponding Artin-Tits groups (Crisp-Paris, 2004).

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- Let $N \trianglelefteq B(W_{B_n})$ be the smallest normal subgroup containing $\mathbf{s}_0^2$, where $s_0 \in S_{B_n}$ is the reflection along the short root. Set $\overline{B(W_{B_n})} := B(W_{B_n})/N$. Then Allcock (2002) noticed that $B(W_{D_n})$ can be realized as an index two subgroup of $\overline{B(W_{B_n})}$.

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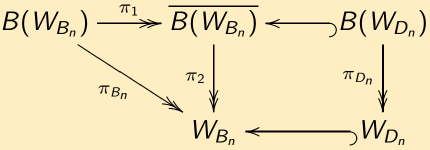
Lemma

There is a commutative diagram



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The Mikado braids of type D_n viewed inside $\overline{B(W_{D_n})}$ are precisely the images of those Mikado braids of type B_n which surject onto elements of W_{D_n}, that is, we have

$$\mathrm{Mik(W_{D_n})} = \{\pi_1(\beta) \mid \beta \in \mathrm{Mik(W_{B_n})} \textit{ and } \pi_{\mathrm{B_n}}(\beta) \in \mathrm{W_{D_n}}\}.$$

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- The proof that simple dual braids are Mikado braids in types A_n and B_n is topological: for $x \in P_c$, starting from the corresponding noncrossing partition one associates to it an Artin braid representing $i_c(x)$. This Artin braid is then topologically shown to be Mikado.

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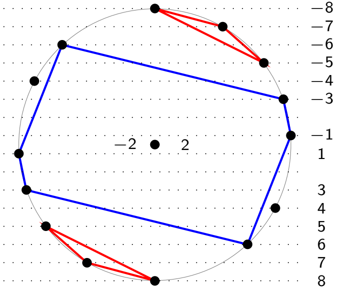

FIGURE 3: A noncrossing partition x in W_{D_n}

Simple dual braids are Mikado braids

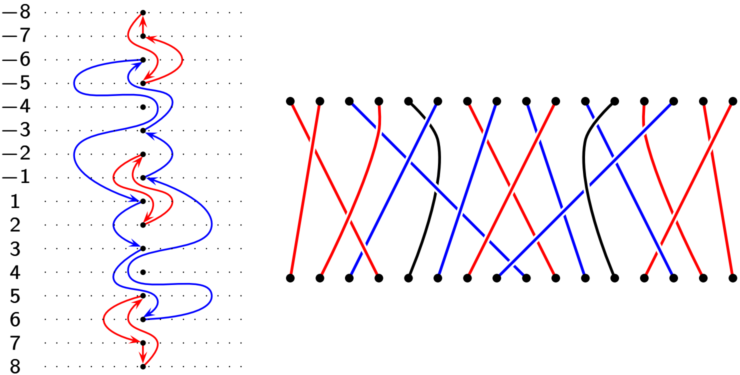

FIGURE 4: The simple dual braid corresponding to the noncrossing partition x. The braid on the right is a type B_n ($n = 8$) braid which is Mikado. Its image in $B(W_{B_n})$ is the simple dual braid corresponding to x. It is Mikado in $B(W_{D_n})$ since its preimage in $B(W_{B_n})$ is.

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- Is there a nice algebraic description of the embedding $B_c^* \hookrightarrow B(W)$? Are there uniform formulas for simple dual braids viewed inside $B(W)$?
- Can we relate Mikado braids to simple dual braids in Artin groups of non-spherical types where dual braid monoids exist (free groups, Artin groups of affine type $\widetilde{A}_n$ and $\widetilde{C}_n$, ...)?

Thank you !

And happy “birthday” Patrick !