Coxeter sortable elements and dual braid monoids

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Let $A \subseteq G$ as above, and let $g \in G$ be balanced. If $A \subseteq P$ and the posets $(P, \leq_l)$, $(P, \leq_r)$ are lattices, then M is a (quasi-)Garside monoid. As a consequence, the word problem in $G(M)$ is solvable.

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Coxeter sortable elements and dual braid monoids

Theorem (Reading)

Let w be c-sortable, let $\mathrm{Cov}(w) := wD_R(w)w^{-1}$. There is a unique $u \in P_c$ such that $\langle \mathrm{Cov}(w) \rangle = \langle t \in T \mid t \leq_T u \rangle$. The assignment $w \mapsto \mathrm{nc}_c(w) := u$ defines a bijection between the set of c-sortable elements and P_c.

Theorem (G.)

Let $x \in P_c$. Let $y = \mathrm{nc}_c^{-1}(x^{-1}c)$. Then $\mathbf{x} = x_{N(y)}$.

- It gives a canonical expression for any simple dual braid in terms of the classical generators of B, and gives a new proof of the fact that every $\mathbf{x} \in \mathbf{P}_c$ we be written in the form $u_\emptyset v_\emptyset^{-1}$ (Digne-G., Baumeister-G., Licata-Queffelec).