Abstracts

Davide Barilari : *Geometry in small time heat kernel expansion for hypoelliptic Hörmander operators.*
Abstract : In this talk I will survey some results about the link between the analytical properties of the heat kernel and the geometry associated with an hypoelliptic second order operator of Hörmander type. In particular I will focus on the small time heat kernel expansion at the cut locus and on the diagonal, presenting classical and recent results, and some conjectures/open problems.

Ugo Boscain : *Sub-Riemannian Random walks and the correponding Laplacians.*
Abstract : We relate some constructions of stochastic analysis to differential geometry, via random walk approximations. We consider walks on both Riemannian and sub-Riemannian manifolds in which the steps consist of travel along either geodesics or integral curves associated to orthonormal frames, and we give particular attention to walks where the choice of step is influenced by a volume on the manifold. A primary motivation is to explore how one can pass, in the parabolic scaling limit, from geodesics, orthonormal frames, and/or volumes to diffusions, and hence their infinitesimal generators, on sub-Riemannian manifolds, which is interesting in light of the fact that there is no completely canonical notion of sub-Laplacian on a general sub-Riemannian mani- fold. However, even in the Riemannian case, this random walk approach illuminates the geometric significance of Ito and Stratonovich stochastic differential equations as well as the role played by the volume.

Rémi Buffe : *Carleman estimate in the neighborhood of a multi-interface.*
Abstract : Carleman estimates are important tools in PDE's, in particular for unique continuation properties, control theory or inverse problems. These estimates are energy estimates of the form

$$\tau^\alpha ||e^{\tau\varphi}u||_{L^2} \lesssim ||e^{\tau\varphi}Pu||_{L^2},$$

where $\tau > 0$ is a semi-classical large parameter, φ is the weight function, and P an operator. We shall establish such an estimate in a neighborhood of an interface where n manifolds of dimension d meet, for elliptic operators of any order. Such situations naturally appear for instance in the case of networks (i.e $d = 1$). This work is a generalization of existing results at a boundary or at a classical interface ($n = 2$).

Laurent Charles : *Quantum speed limit vs displacement energy.*
Abstract : The quantum speed limit is a universal bound on the energy required to pass from one state to another orthogonal state in a quantum system. Similarly, in symplectic topology, the displacement energy is the minimal energy needed to displace a given subset of a symplectic manifold. I will discuss how these two notions are related in the semiclassical limit. Joint work with Leonid Polterovich.

Suresh Eswarathasan : *Scarring of Quasimodes on Hyperbolic Manifolds.*
Abstract : Let M be a compact Riemannian manifold. There is a classical result in microlocal analysis which states that given an elliptic periodic orbit $\gamma \subset S^*M$ with invariant measure δ_γ, we can construct Laplace-Beltrami quasimodes of order $\mathcal{O}(h^\infty)$ whose semiclassical measure is exactly δ_γ (due to Colin de Verdière, Ralston, and others). In the case of hyperbolic periodic orbits, the well-known Gaussian beam construction used to construct these quasimodes breaks down.

For a compact hyperbolic manifold (M, g) and a totally geodesic submanifold N (or rather a measurable, flow-invariant subset $\Gamma \subset S^*M$ with an invariant probability measure μ_Γ), we construct logarithmic quasimodes (i.e. those of order $\epsilon h/|\log h|$ for a given $\epsilon > 0$) whose semiclassical measure contains δ_{S^*N} (or more generally μ_Γ) in its ergodic decomposition. This generalizes previous results of Brooks and E.-Nonnenmacher concerning periodic orbits on compact surfaces. This is joint work with Lior Silberman (U. British Columbia, Vancouver, Canada).

Lysianne Hari : *Propagation of coherent states.*
Abstract : We study the propagation of a coherent state for systems of coupled nonlinear Schrödinger equations in the semi-classical limit. We will consider different cases, leading to different physical phenomena.

First, couplings will be induced by a cubic nonlinearity and some stability of the solution will be studied : an initial coherent state polarized along an eigenvector of the potential remains - at leading order in the same eigenspace ("adiabaticity"). Then we will add a linear coupling, thanks to a matrix-valued potential presenting an "avoided crossing" at one given point : the gap between its eigenvalues reduces as the semi-classical parameter becomes smaller. We will show that when an initial coherent state polarized along an eigenvector of the potential propagates through the avoided crossing point, there are transitions between the modes at leading order, whose probabilities are given by the linear Landau-Zener formula. We will finally handle a "smooth" exact crossing, and see that one can proceed as in the linear case to study the propagation in our setting. A comparison with linear results will be made in each case.

Alexey Kokotov : *DtN isospectrality, flat metrics with non-trivial holonomy and comparison formula for determinants of Laplacian.*
Abstract : We study comparison formulas for ζ-regularized determinants of self-adjoint extensions of the Laplacian on flat conical surfaces of genus $g \geq 2$. The cases of trivial and non-trivial holonomy of the metric turn out to differ significantly. This is the joint work with Luc Hillairet (Orleans).

Camille Laurent : *Quantitative unique for operators with partially analytic coefficients. Applications to Control.*
Abstract : In this talk, I will describe some result with Matthieu Léautaud where we give logarithmic estimates that quantify some known unique continuation results for operators with coefficients analytic in some part of the variable. We will give some applications to control theory.

Yohann Le Floch : *Comparing classical quantum states in the semiclassical limit in Kähler quantization.*
Abstract : After briefly reviewing the quantization of compact Kähler manifolds, I will describe a procedure to construct states concentrating on submanifolds with densities, analogous to the so-called classical quantum states in the physics literature. These are mixed states, and I will explain how to quantify how far they are from being pure. I will also explain how one can try to compare two such states by means of their fidelity and other related quantities.

Fabricio Maciá : *Concentration, non-concentration and controllability of Schrödinger flows.*

Gabriel Rivière : *Spectral analysis of Morse-Smale gradient flows.*
Abstract : Given a function and a Riemannian metric on a compact and boundaryless manifold, one can define a gradient vector field. Under generic assumptions (of Morse-Smale type), I will explain how one can compute explicitely the spectrum of the associated Lie derivative acting on certain anisotropic Sobolev spaces of currents. As a motivation, I will give some applications to dynamical systems (decay of correlations) and to differential topology (spectral interpretation of the Morse complex).

This is a joint work with Nguyen Viet Dang (University Lyon 1).

Luca Rizzi : *Quantum confinement on non-complete Riemannian manifolds.*
Abstract : We consider the quantum completeness problem, i.e. the problem of confining quantum particles, on a non-complete Riemannian manifold M equipped with a smooth measure, possibly degenerate or singular near the metric boundary of M, and in presence of a locally square integrable real-valued potential V. We identify an intrinsic object, the effective potential, which allows to formulate simple criteria for quantum confinement. These criteria allow us to : (i) obtain quantum confinement results for measures with degeneracies or singularities near the metric boundary of M ; (ii) generalize the Kalf-Walter-Schmincke-Simon Theorem for strongly singular potentials to the Riemannian setting for any dimension of the singularity ; (iii) give the first, to our knowledge, curvature-based criteria for self-adjointness of the Laplace-Beltrami operator ; (iv) prove, under mild regularity assumptions, that the Laplace-Beltrami operator in almost-Riemannian geometry is essentially self-adjoint, partially settling a conjecture formulated in [Boscain, Laurent - Ann. Inst. Fourier, 2013]. (joint work with D. Prandi and M. Seri)

Henrik Ueberschär : *Seba's billiard and the random wave model.*
Abstract : Seba's billiard, a rectangle with a Dirac mass placed in the interior, is a popular model in Quantum Chaos to study the transition between integrable and chaotic dynamics in quantum systems. A conjecture by Seba suggests that Berry's random wave model is valid for such systems and as a consequence the value distribution of its wave functions converges to a Gaussian in the semi-classical limit. I will explain why, in the generic case, this conjecture is false. This is joint work in progress with Par Kurlberg (KTH Stockholm).

Martin Vogel : *Spectral statistics of non-normal operators subject to small random perturbations.*

Abstract : It is well known that the spectrum non-normal operators can be highly unstable even under tiny perturbations. Exploiting this phenomenon it was shown in recent works by Hager, Bordeaux-Montrieux, Sjöstrand, Christiansen and Zworski that one obtains a probabilistic Weyl law for a large class of non-normal semiclassical pseudo-differential operators after adding a small random perturbation. We will discuss some recent results obtained in collaboration with Stéphane Nonnenmacher concerning the local statistics of eigenvalues of such operators. That is the statistical interaction between the eigenvalues on the scale of their average spacing.