# Resolvent analysis of subwavelength resonances

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We consider the propagation of acoustic waves in the presence of inhomogeneities having small size and high contrast with respect to a regular background. When the size and the contrast of the acustic micro-bubbles have a suitable asymptotics and the incident wave has specific proper frequencies, such *composite systems* exhibit a transition towards a resonant regime where an enhancement of the scattered wave is observed. The resonant frequencies, depending on the capacitance of the perturbation, are usually referred to as *subwavelength resonances*, since the corresponding wavelength can be several orders of magnitude larger than the size of the inhomogeneities. Such a resonant regime has tremendous applications in imaging, in the broad sense, and material sciences, to cite a few.

In the case where a single micro-bubble of size $\varepsilon$ enjoyes a high contrast of both its mass density and bulk modulus, the resonant frequency is referred to as the *Minnaert frequency* $\omega_M$. Here we explain the origin of this phenomenon: at first we show that the scattering of an incident wave of frequency $\omega$ is described by a self-adjoint $\omega$-dependent Schrödinger operator with a singular potential supported at the inhomogeneity interface. Then, we consider its norm-resolvent limit in the low-frequency regime (corresponding in our setting to $\varepsilon \ll 1$). We prove that this limit is non-trivial – i.e., the frequency-dependent operator asymptotically differs from the Laplacian – if and only if $\omega = \omega_M$.

The limit operator describing the non-trivial scattering process is explicitly determined and belongs to the class of point perturbations of the Laplacian. When the frequency of the incident wave approaches $\omega_M$, the scattering process undergoes a transition between an asymptotically trivial behaviour and a non-trivial one. At each frequency $\omega$, we provide an explicit asymptotic expansion of the scattered field as $\varepsilon \to 0$. The resolvent-approach naturally yields a uniform-in-space control of the error terms, improving in this sense previous results.

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