Cosmology of models with spontaneous scalarization: instability and a cure

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Theoretical aspects of modern cosmology
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Outline

- **Motivation**
- **What is scalarization**
- **Cosmological instability of scalar-Gauss-Bonnet theories exhibiting scalarization**
- **Reconciling spontaneous scalarization with cosmology**
- **Conclusions**

Motivation

- **Testable gravity modifications: neutron stars and black holes**
- **Benchmarks for testing General Relativity:**
  - LIGO/Virgo
  - LISA
  - EHT
  - GRAVITY
  - …

Original model of scalarization by Damour-Esposito-Farèse

Standard scalar-tensor theory:

$$S_{\mathrm{DEF}} = \int \mathrm{d}^4x \frac{\sqrt{-g}}{16\pi G} \left(R - 2\partial_\mu\varphi\partial^\mu\varphi\right) + S_{\mathrm{m}}\left[A^2(\varphi)g_{\mu\nu}, \psi_{\mathrm{m}}\right]$$

$$\kappa = 8\pi G$$

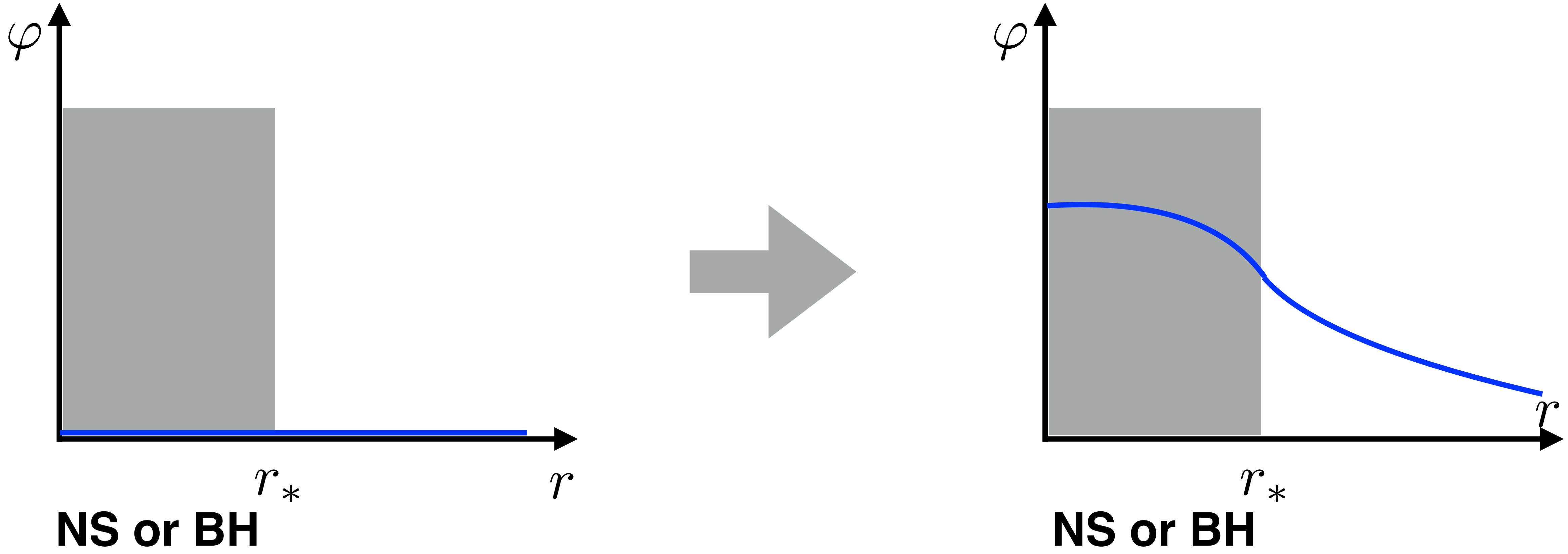
$$A(\varphi) = e^{\frac{1}{2}\beta\varphi^2}$$

β is a constant, which feeds into deviations from GR

φ is dimensionless

The idea of scalarization

Profile of the scalar field:



Original model of scalarization by Damour-Esposito-Farèse

[Damour&Esposito-Farese'93'96]

Essence of scalarization:

$$\Box\varphi + 4\pi G\alpha(\varphi)T^{\mathrm{m}} = 0$$

$$\alpha(\varphi) \equiv \frac{\mathrm{d}\ln A(\varphi)}{\mathrm{d}\varphi} = \beta\varphi$$

$$T^{\mathrm{m}}_{\mu\nu} = \frac{2}{\sqrt{-g}}\frac{\delta S_{\mathrm{m}}}{\delta g^{\mu\nu}}$$

- $\varphi = 0$ is the GR solution;
- For $\beta < 0$ and $T^{\mathrm{m}} > 0$, tachyonic mass $\Rightarrow$ other solutions?

This is indeed the case for $\beta \lesssim -4$ for neutron stars

Original model of scalarization by Damour-Esposito-Farèse

- No scalarization for the Sun and the Earth

- Scalarized neutron stars

Not much deviation from GR in Solar system: check PPN parameters:

[Damour&Esposito-Farese'92]

$$\gamma_{\rm PPN}-1=\frac{-2\alpha^2(\varphi_0)}{1+\alpha^2(\varphi_0)}\qquad \beta_{\rm PPN}-1=\frac{\beta\alpha^2(\varphi_0)}{[1+\alpha^2(\varphi_0)]^2}$$

In the limit $\alpha(\varphi_0)\to 0$, the PPN parameters coincide with those of GR.

Scalar-Gauss-Bonnet model of scalarization

[Doneva&Yazadjiev'17
Silva et al'17
Antoniou et al'17]

$$S=\int \mathrm{d}^4x\sqrt{-g}\left[\frac{M_P^2}{2}R-\frac{1}{2}g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi+f(\varphi)\hat{G}\right]$$

$$\hat{G}=R_{\mu\nu\sigma\alpha}R^{\mu\nu\sigma\alpha}-4R_{\mu\nu}R^{\mu\nu}+R^2$$

$$\Box\varphi+f'(\varphi)\hat{G}=0 \qquad \Longleftrightarrow \qquad \Box\varphi+4\pi G\alpha(\varphi)T^{\mathrm{m}}=0$$

f(phi)-GB model

Vanilla DEF model

When $f'(\varphi_0)=0$ and $f''(\varphi_0)>0$ then black holes and neutron stars are spontaneously scalarized

Instability of the GR solution

Trivial scalar $\phi_{\mathrm{GR}} = 0$ and background Schwarzschild metric

$$\Box\varphi + f'(\varphi)\hat{G} = 0$$

Perturbations:

$$\left(\Box + f''(\varphi_0)\hat{G}\right)\delta\varphi = 0$$

Schrödinger-like equation:

$$\frac{\mathrm{d}^2 u}{\mathrm{d}r_*^2} + \omega^2 u = V_{\mathrm{eff}}(r)u$$

$$V_{\mathrm{eff}}(r) = \left(1 - \frac{r_g}{r}\right)\left(\frac{r_g}{r^3} - \frac{12r_g^2}{r^6}f''(\varphi_0)\right)$$

Instability of the GR solution

A sufficient condition for the existence of an unstable mode

$$\int_{r_g}^{\infty} \mathrm{d}r \frac{V_{\mathrm{eff}}(r)}{1-\frac{r_g}{r}} < 0 \quad \Rightarrow \quad r_g^2 < \frac{24}{5} f''(\varphi_0)$$

$f(\varphi) \ = \ \frac{1}{8}\lambda^2\varphi^2 + \lambda^2\mathcal{O}\left(\varphi^4/M_1^2\right)$, normally $M_1 \sim M_P$

Scalarization happens for $\lambda \gtrsim r_g$, i.e. $\lambda \sim \frac{M_\odot}{M_P^2}$

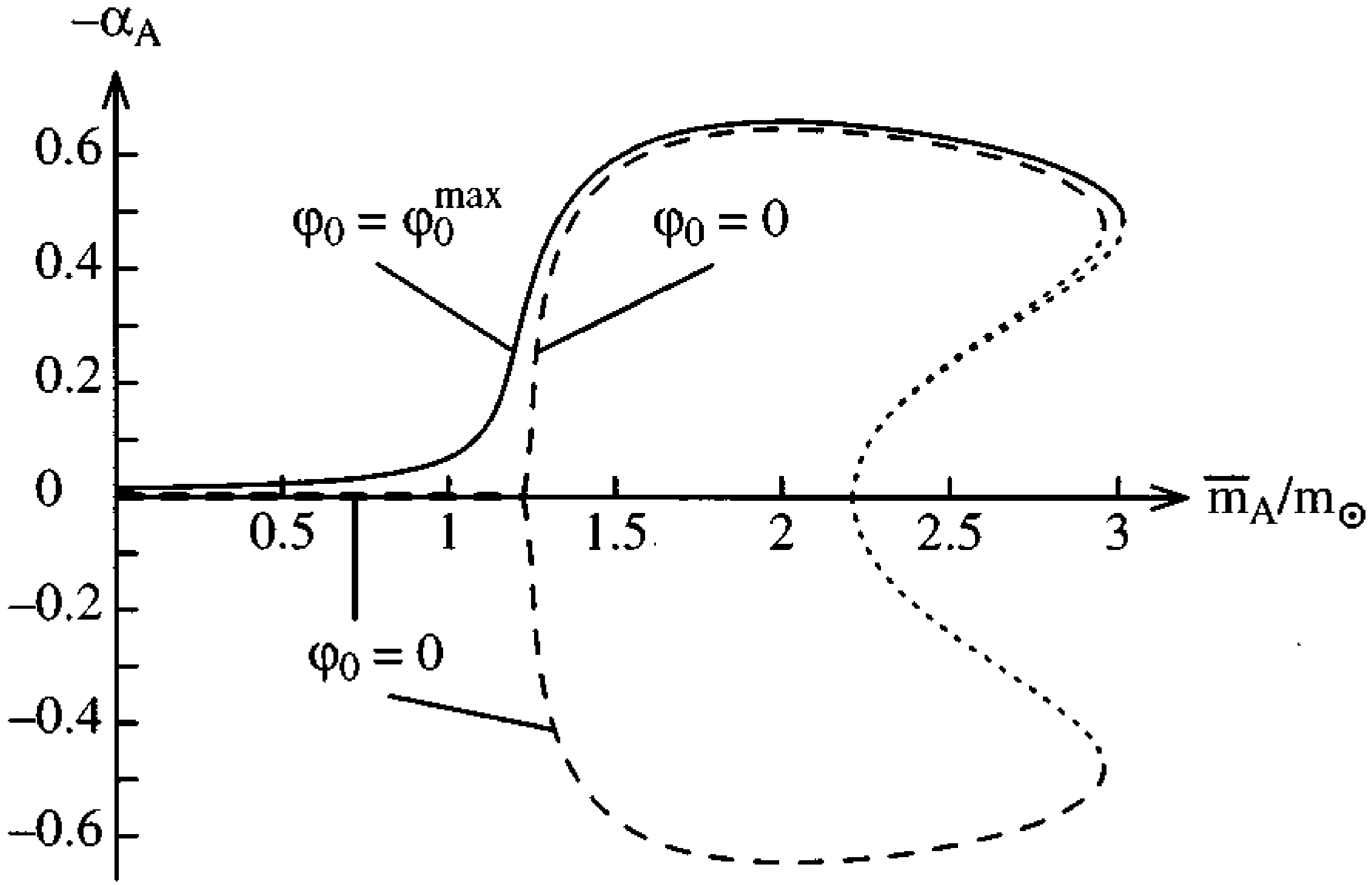
Similarly in DEF model:

$$\Box\delta\varphi + \frac{\kappa}{2}\beta T^{\mathrm{m}}\,\delta\varphi = 0$$

For $\beta \lesssim -4$ the GR solution for neutron stars, $\varphi_{GR} = 0$, is unstable

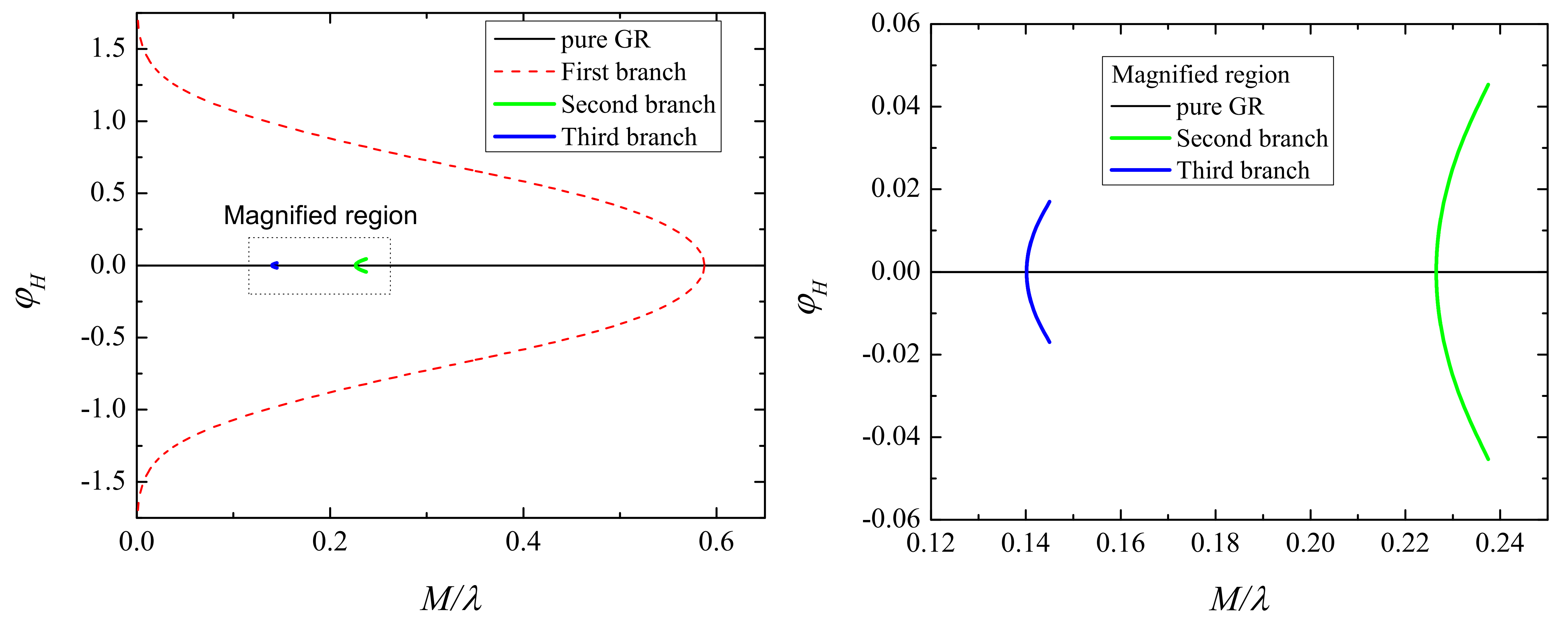
Original model of scalarization by Damour-Esposito-Farèse

[Damour&Esposito-Farese'96]



Scalar-Gauss-Bonnet model of scalarization

[Doneva&Yazadjiev'17]

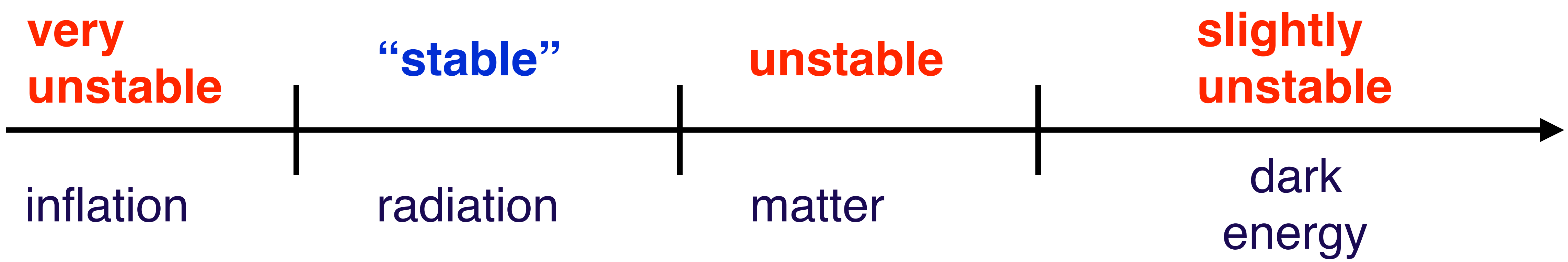


1. **Is tachyonic instability present in cosmology?**
2. **Is it dangerous?**

instability in cosmology

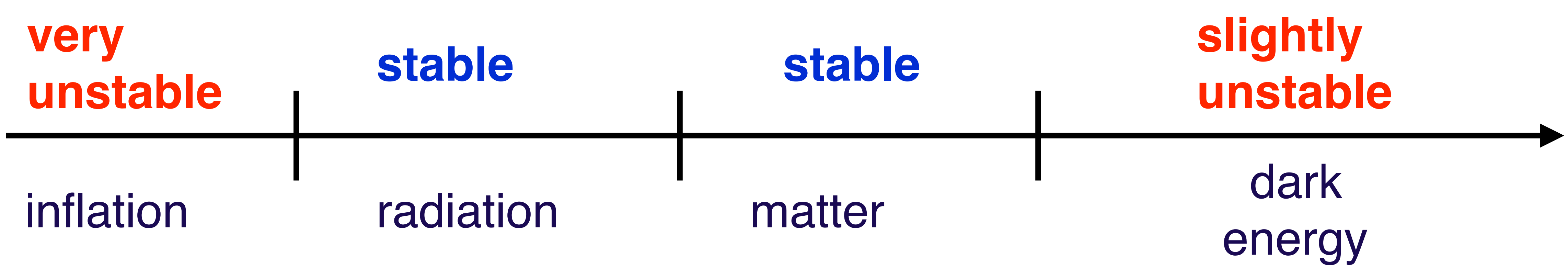
DEF model

$$\Box\varphi + \frac{\kappa}{2}\beta T^{\mathrm{m}}\varphi = 0$$



Scalar-GB model

$$\left[\Box + f''(\varphi_0)\hat{G}\right]\delta\varphi = 0 \qquad m_{\mathrm{eff}}^2 = -6\lambda^2 H^2 \frac{\ddot{a}}{a}$$



constraint in DEF model

Shapiro time-delay measurement: $\gamma_{\mathrm{PPN}} = 1 \pm (2.1 \pm 2.3) \times 10^{-5}$

$$\Downarrow$$

$$\varphi_0 \lesssim 10^{-3}$$

Can we "naturally" have a small value for the cosmological scalar field?

instability in DEF model

Matter domination instability: $\varphi_{\rm eq} \lesssim 10^{-10}$

Radiation domination instability: $\dfrac{\varphi_{\rm eq}}{\varphi_{\rm i}} \simeq 10$

At the end of inflation we should have $\varphi_{\rm i} \lesssim 10^{-11}$

instability in DEF model during inflation

Inflation: Fine-tune $\varphi = 0$ in the beginning of inflation.
Perturbations:

$$\langle(\delta\varphi)^2\rangle_{k\in\{k_{min},k_{max}\}} = \frac{2^{2\nu}\Gamma^2(\nu)}{2(2\nu-3)\pi^2} \cdot \frac{H^2}{M_{Pl}^2} \cdot \left|\frac{\eta_*}{\eta}\right|^{2\nu-3} \gg 1$$

η_* is the time when the cosmological mode with wavenumber $k_{\min}$ exits horizon

$k_{\min} \simeq H_0 \qquad \frac{\eta_*}{\eta} \sim e^{60}$

attempts to solve the instability problem

[de Pirey Saint Alby & Yunes'17]

Add potential $V(\varphi) = \frac{m^2\varphi^2}{2}$:

at the moment $H \sim m$ the field starts to oscillate contributing to DM.

The assumptions $\varphi_{\rm i} \simeq 1$ and $\dot{\varphi}_{\rm i} \simeq 0$ and the mass $m \lesssim 10^{-28}$ eV.

Otherwise the contribution to the energy density is too large

The instability during inflation gives $\varphi_{\rm i} \gg 1$

attempts to solve the instability problem

[Anderson et al'16]

$$\ln A(\varphi) = \frac{\beta\varphi^2}{2} \to \frac{\beta\varphi^2}{2} + \frac{\lambda\varphi^4}{4}$$

For $\lambda > 0$ the field φ is stabilized during inflation, so that $\varphi = \sqrt{-\beta/\lambda}$ up to now.

This scenario does not work, because with the scalarization of neutron stars does not occur.

Curing DEF model

idea: Give large normal mass to the scalar responsible for scalalrization during inflation and kill the mass after inflation ends

$$\sim \varphi^2 \chi^2$$

$$S_{\rm DEF} = \int {\rm d}^4 x \frac{\sqrt{-g}}{16\pi G} \left(R - 2\partial_\mu \varphi \partial^\mu \varphi \right) + S_{\rm m} \left[A^2(\varphi) g_{\mu\nu}, \psi_{\rm m} \right]$$

$$V(\varphi, \chi) = g^2 \varphi^2 \chi^2$$

We assume $m_{\rm eff}^2 = g^2 \chi^2 \gg H^2$

For $\chi \simeq M_{Pl}$ and $H \simeq 10^{13}$ GeV, $g^2 > 10^{-12}$

Curing DEF model

background:

$$\ddot{\varphi} + 3H\dot{\varphi} + m^2\varphi = 0$$

$$m^2 = g^2\chi^2 + 6\beta H^2$$

$$\varphi = \frac{C}{a^{3/2}} \cdot \cos\left[\sqrt{m^2 - \frac{9H^2}{4}}\, t + \delta\right]$$

Starting from $\varphi \simeq 1$ by the end of inflation the field φ is relaxed to $\boxed{\varphi \lesssim 10^{-39}}$

Curing DEF model

Perturbations:

$$\delta\ddot{\varphi}_{\mathbf{k}} + 3H\delta\dot{\varphi}_{\mathbf{k}} + \frac{k^2}{a^2}\delta\varphi_{\mathbf{k}} + \frac{\kappa}{2}\frac{\partial\alpha(\varphi)}{\partial\varphi}T^{\mathrm{m}}\delta\varphi_{\mathbf{k}} + \frac{1}{2}\frac{\partial^2 V}{\partial\varphi^2}\delta\varphi_{\mathbf{k}} = 0$$

damped oscillator with an almost constant large mass;

$\delta\varphi_{\mathbf{k}}$ decay as $\frac{1}{a^{3/2}}$ in the superhorizon regime

$$\langle\delta\varphi^2\rangle_{\text{unstable}}(\eta_f) = \frac{H^2}{3\pi s M_{Pl}^2}\cdot\left|\frac{\eta_f}{\eta_\times}\right|^3 \Rightarrow \boxed{\sqrt{\langle\delta\varphi^2\rangle_{\text{unstable}}(\eta_f)} \simeq 10^{-46}}$$

$$s = \sqrt{\frac{m^2}{H^2} - \frac{9}{4}} \qquad \frac{\eta_f}{\eta_\times} \sim e^{-60}$$

Curing DEF model

NB.

1) Matter-like (p)reheating does not affect the result

2) Trace anomaly is subdominant in the radiation stage

No Cure for scalar-Gauss-Bonnet

Vanilla scalar-tensor theory:

$$S_{\mathrm{DEF}} = \int \mathrm{d}^4x \frac{\sqrt{-g}}{16\pi G}\left(R - 2\partial_\mu\varphi\partial^\mu\varphi\right) + S_{\mathrm{m}}\left[A^2(\varphi)g_{\mu\nu}, \psi_{\mathrm{m}}\right]$$

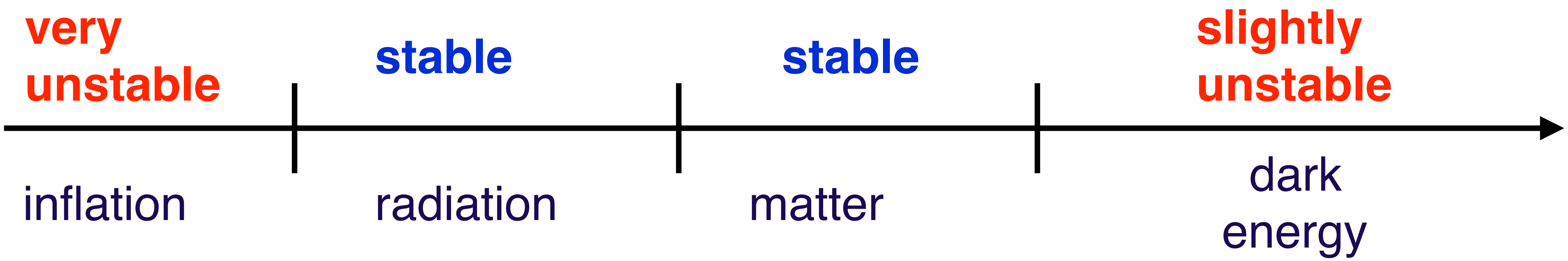
$A(\varphi) = e^{\frac{1}{2}\beta\varphi^2}$ $\quad$ β is a constant, which feeds into deviations from GR

Scalar-Gauss-Bonnet theory

$$S = \int \mathrm{d}^4x\sqrt{-g}\left[\frac{M_P^2}{2}R - \frac{1}{2}g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi + f(\varphi)\hat{G}\right]$$

$f(\varphi)\hat{G} = \frac{1}{8}\lambda^2\varphi^2\hat{G} + \ldots$ $\quad$ $\lambda \sim \frac{M_\odot}{M_P^2} \sim 10^{19}\mathrm{GeV}^{-1}$

No Cure for scalar-Gauss-Bonnet



$$\frac{t_{\rm inst}}{t_{\rm inf}} \sim \frac{1}{N\lambda H_{\rm inf}} \sim 10^{-34}$$

for inflation with the scale
$H_{\rm inf} \sim 10^{13}\text{GeV}$ and $N \sim 10^2$ e-foldings

$$\frac{t_{\rm inst}}{t_0} \sim \frac{H_0}{m_{\rm eff}} \sim \frac{1}{\lambda H_0} \sim 10^{23}$$

No Cure for scalar-Gauss-Bonnet

$$\rho_{\mathrm{GB}} = -6\lambda^2 H^3 \varphi\dot{\varphi}$$

$$\langle \phi(\mathbf{x},\eta)\frac{\partial}{\partial\eta}\phi(\mathbf{x},\eta)\rangle = \frac{H^2}{8\pi^2\nu|\eta|}\cdot\left(\frac{2\nu}{e}\right)^{2\nu}\cdot\left[\left|\frac{\eta_1}{\eta}\right|^{2\nu-3}-1\right]$$

$$\nu = \sqrt{\frac{9}{4}-\frac{m_{\mathrm{eff}}^2}{H^2}} \sim 10^{32}$$

Destabilisation occurs very quickly => no standard cosmological inflation

Trying to cure scalar-Gauss-Bonnet scalarization model

$$f(\phi) \;=\; \frac{1}{8}\lambda^2\phi^2 + \lambda^2\frac{\phi^4}{M_P^2}$$

Destabilisation happens before the quartic correction kicks in

$$\langle\phi^2(\mathbf{x},\eta)\rangle \simeq \frac{M_P^2}{\lambda^3 H^3}$$

$$\sqrt{\langle\phi^2(\mathbf{x},\eta)\rangle} \sim 10^{-48} M_P \lll M_P$$

Trying to cure scalar-Gauss-Bonnet scalarization model

The same idea with coupling to inflaton? $\sim g^2\chi^2\phi^2$

However: $$g^2 \gtrsim \frac{6\lambda^2 H^4}{\chi^2} \simeq 10^{53}$$

for $\chi \simeq M_P$, $\lambda H \simeq 10^{32}$, and $H \simeq 10^{13}$ GeV

Trying to cure scalar-Gauss-Bonnet scalarization model

extra coupling of the scalar to higher powers of curvature: $\sim \phi^2 R^4$

Corresponds to extra scalar d.o.f. a la f(R)

The coupling of this extra scalar to the "scalarization" scalar should be huge as in the previous example

Trying to cure scalar-Gauss-Bonnet scalarization model

quartic terms in the scalar: $\sim \phi^4 \hat{G}$

Destabilization happens long before the correction takes place

Add large coupling? $\sim \tilde{g}\phi^4 \hat{G}$

The scalarized solution would be indistinguishable from GR solution

Trying to cure scalar-Gauss-Bonnet scalarization model

[Macedo et al'19]

quartic self-interaction: $\sim g\phi^4$

the scalar field is in the minimum of the effective potential during inflation

However: $\phi_{min} \sim 10^{64} M_P$

Also extra potential energy: $\sim 10^{180} M_P^4$

Conclusions

- Old time DEF and modern scalar-Gauss-Bonnet models of scalarization gives interesting modifications of gravity, leading to different from GR neutron stars or/and black hole solutions.
- Both models do contain dangerous instabilities during inflation
- It is possible to `naturally' cure the original DEF model
- It seems however that the cure of the scalar-Gauss-Bonnet model is beyond hope.