Global Properties of the Growth Index of Matter Inhomogeneities in the Universe

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Phys. Rev. D 100, 083503 (2019)

Outline

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- Formalism : The Growth Index γ
- Results
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  - Varying Equation of State $w_{DE}(a) = w_0 + w_a(1 - a)$
  - Beyond GR: A bump in $G_{\text{eff}}(a, k)$
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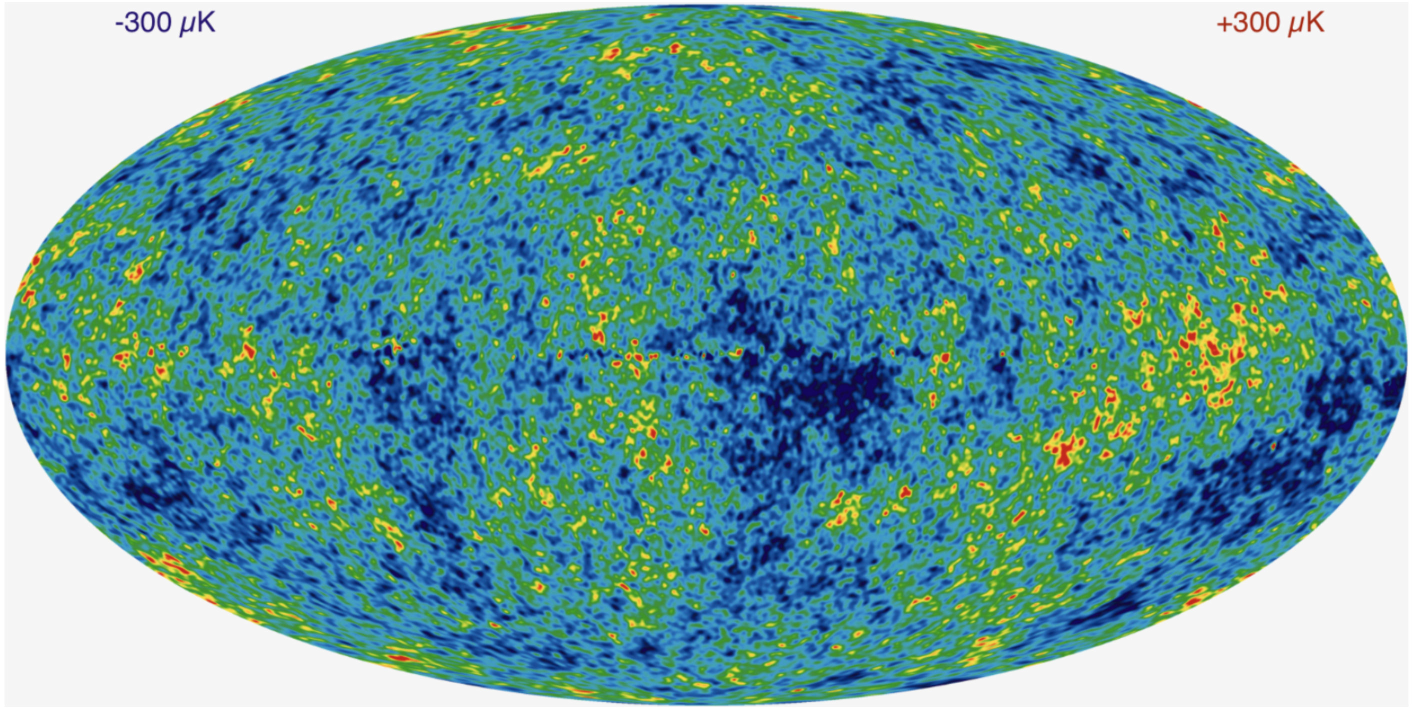
Motivations

→ CMB data in perfect agreement with Gaussian, (nearly) Scale-Invariant Perturbations generated by the Inflaton ϕ



[Planck '15]

→ Tensions within the ΛCDM paradigm suggest Λ (or GR) might not be the end of the story

→ LSSF is highly-dependent on the nature of Dark Energy / Laws of gravity

One of our best probes, Discrimination between Models

→ Trace back the evolution of those seeds in time, and try to infer something about the background's expansion $H(z)$ from $\delta(z)$

→ Next generation of data (Euclid & WFIRST)

Assumptions

- We place ourselves long after the RD Epoch ($z \ll z_{eq}$)
  + Flat Universe ($\Omega_k \sim 0$) → Insured by Inflation!

$$h^2(z) = \frac{H^2(z)}{H_0^2} = \Omega_{m,o}(1+z)^3 + (1-\Omega_{m,0}) \exp\left[3\int_0^z dz' \frac{1+w_{DE}(z')}{1+z'}\right]$$

- Newtonian Framework valid for dust, non-relativistic matter (DM & Baryons) deep inside the Hubble Radius ($k \gg aH$)

→ GR treatment includes corrections $\mathcal{O}\left(\frac{a'}{a}\right)^2 \sim a^2H^2$ (dilation terms) which are negligible inside the Horizon

- We only consider models where DE does not cluster! $\delta\rho_{DE} \sim 0$
  (Valid for Λ, Quintessence and non-interacting DE)

- As usual $\delta \ll 1$, as soon as $\delta\rho_m \sim \bar{\rho}_m$ the scale becomes non-linear and our formalism breaks down.

Linear Matter Perturbations

$$\frac{d}{dt} = H\frac{d}{d\ln a}$$

$$\ddot{\delta}_m + 2H\dot{\delta}_m = 4\pi G\rho_m\delta_m$$

$$\frac{3}{2}H^2\Omega_m\delta_m$$

$$\delta_m \equiv \frac{\delta\rho_m}{\bar{\rho}_m}$$

"Density Contrast"

Rewritten in terms of a(t):

$$\frac{df}{d\ln a} + f^2 + \left(2 + \frac{\dot{H}}{H^2}\right)f = \frac{3}{2}g\Omega_m\,,$$

$$g(a,k) = G_{\rm eff}/G_N$$

$$f \equiv \frac{d\ln\delta_m}{d\ln a} = \Omega_m(z)^{\gamma(z)}$$

The Growth Function f

The Growth Index γ

$$\gamma(z=0) \simeq 0.55$$

In agreement with ΛCDM

Solving for γ (Ω_m):

Encodes the modification of gravity

$$6w_{DE}\Omega_{DE}\ln\Omega_m\frac{d\gamma}{d\ln\Omega_m} + 3w_{DE}(\Omega_m)\Omega_{DE}(2\gamma-1) + 1 + 2\Omega_m^{\gamma} - 3\,g\,\Omega_m^{1-\gamma} = 0$$

Using Redshift Space Distortions (RSDs)

we can constrain γ

Via the robust observable

r.m.s density fluctuation

$$f\sigma_8(z) \equiv f(z)\cdot\sigma_8(z) = -(1+z)\frac{\sigma_{8,0}}{\delta_0}\delta_m'(z)$$

@scales k=8h⁻¹.Mpc

Evaluated @ Present Time

The Growth Index $\gamma^{\Lambda CDM}$

Setting $w_{DE} = -1$ & $g = 1$

$$-6\Omega_{DE} \ln \Omega_m \frac{d\gamma}{d\ln \Omega_m} - 3\Omega_{DE}(2\gamma - 1) + 1 + 2\Omega_m^{\gamma} - 3\Omega_m^{1-\gamma} = 0$$

$$\boxed{\Omega_{DE} = 1 - \Omega_m}$$

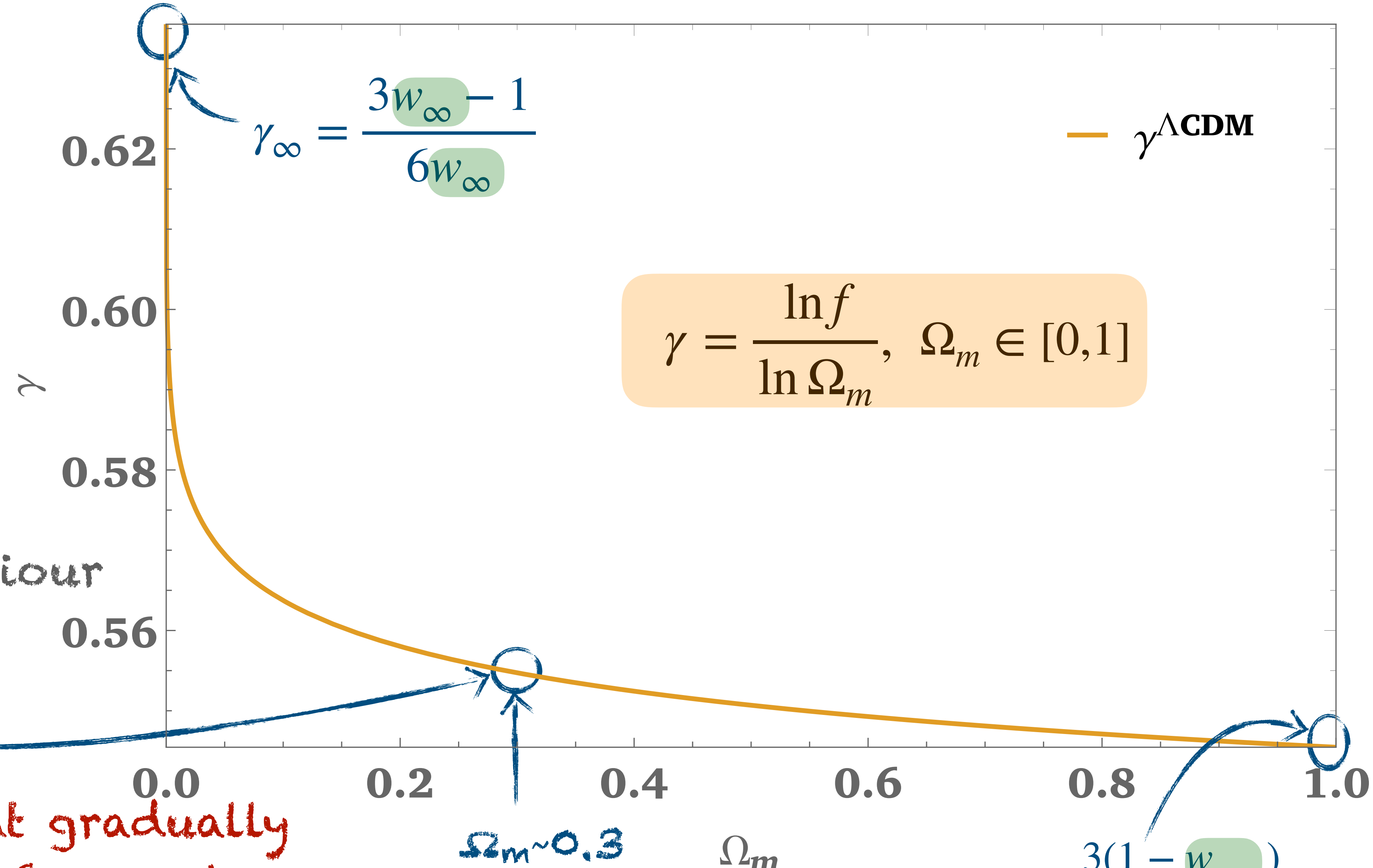
Starting at $\Omega_m = 1$

$$\gamma_{-\infty}^{\Lambda CDM} = \frac{6}{11} \simeq 0.5454$$

Can solve numerically for $\gamma^{\Lambda CDM}$

Nearly constant behaviour

$$\gamma_0^{\Lambda CDM} \simeq 0.55$$



$$\gamma_\infty = \frac{3w_\infty - 1}{6w_\infty}$$

$$\gamma = \frac{\ln f}{\ln \Omega_m}, \quad \Omega_m \in [0,1]$$

$\Omega_m \sim 0.3$

$$\gamma_{-\infty} = \frac{3(1 - w_{-\infty})}{5 - 6w_{-\infty}}$$

As the DE component gradually catches up, the growth of perturbations slows and γ increases rapidly towards the asymptotic value $\gamma_\infty^{\Lambda CDM} = \dfrac{2}{3}$

Notice the Dependence on w_{DE}

The Growth Index $\gamma^{\Lambda CDM}$

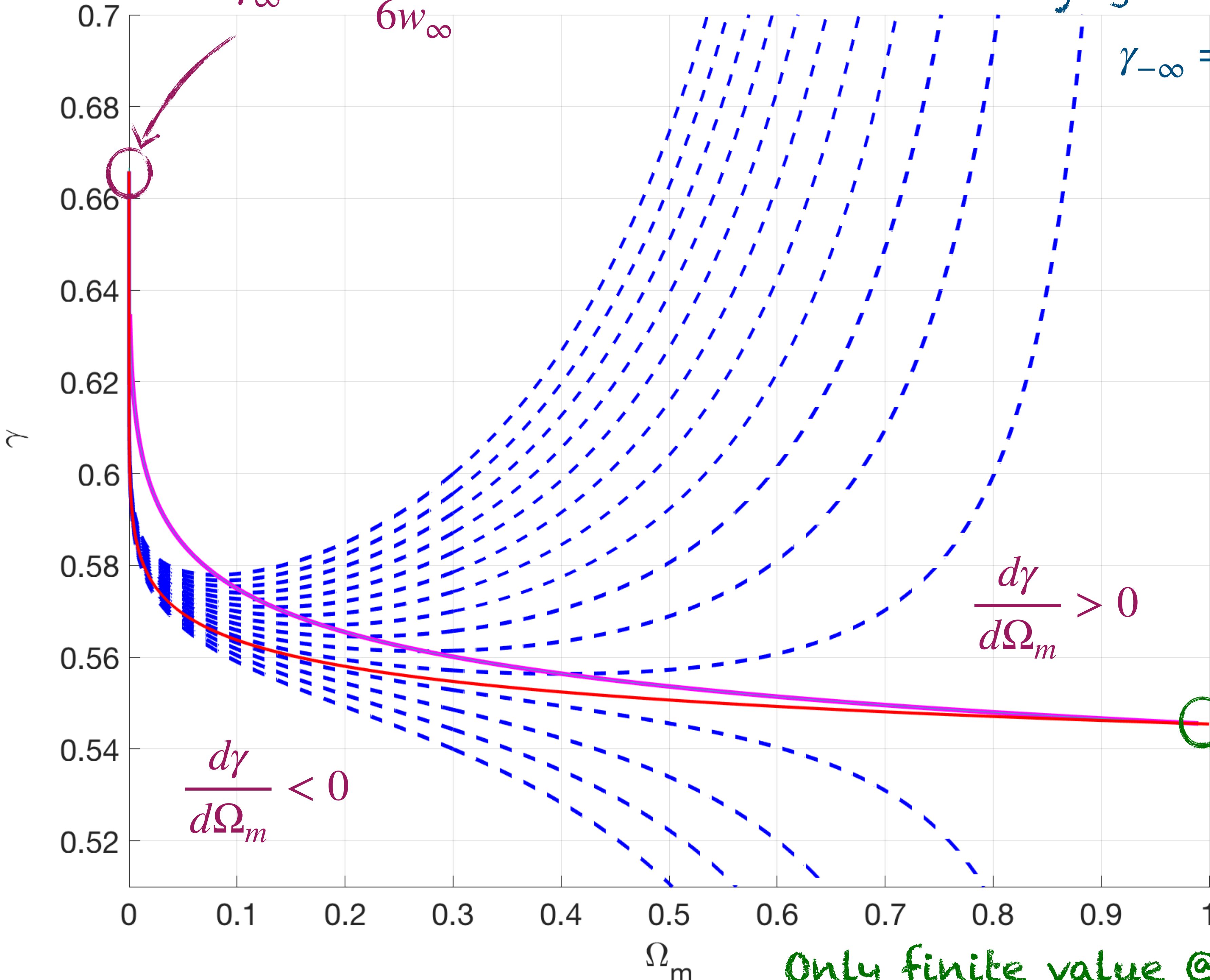
Setting

$w_{DE} = -1$ & $g = 1$

$\gamma_\infty = \dfrac{3w_\infty - 1}{6w_\infty}$

Non-negligible Decaying Mode

$\gamma_{-\infty} = \pm\infty$



$$f = \frac{\delta_1}{\delta} f_1 + \frac{\delta_2}{\delta} f_2$$

In the future

$$f \sim C\, a^{\frac{1}{2}(3w_\infty - 1)} \to 0$$

$$\Omega_m \sim a^{3w_\infty}$$

$$\boxed{\gamma = \frac{\ln f}{\ln \Omega_m}}$$

Function in (Ω_m, γ)-plane Such that:

$$\frac{d\gamma}{d\Omega_m} = 0$$

Only finite value @ $\Omega_m = 1$
Purely growing mode!

Varying EoS for DE

EoS Parameterized by :

$$w_{DE}(a) = w_0 + w_a(1-a)$$

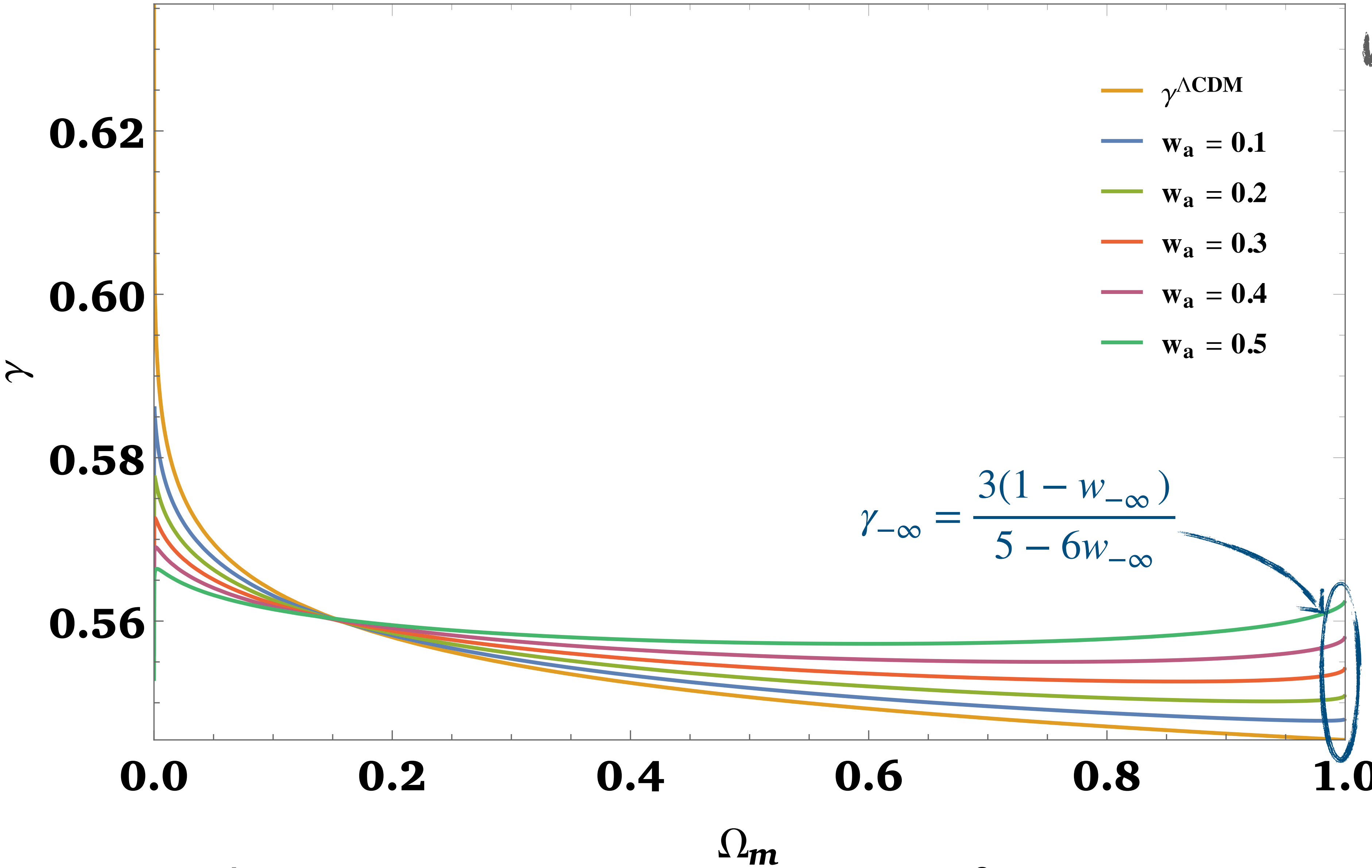
$$6\,w_{DE}(\Omega_m)\ln\Omega_m \frac{d\gamma}{d\ln\Omega_m} + 3\,w_{DE}(\Omega_m)\Omega_{DE}(2\gamma-1) + 1 + 2\Omega_m^{\gamma} - 3\,g\,\Omega_m^{1-\gamma} = 0$$

Using the useful relation:

$$w_{DE} = \frac{1}{3(1-\Omega_m)}\frac{d\ln\Omega_m}{d\ln a}$$

We can solve for a!

$$a(\Omega_m) \to w_{DE}(\Omega_m) \to \gamma(\Omega_m)$$



We restrict ourselves to

$$\boxed{w_{-\infty} = w_0 + w_a < 0}$$

To ensure MD

&

$$\boxed{w_a > 0}$$

To ensure DE Domination

Varying EoS for DE

EoS Parameterized by :

$$w_{DE}(a) = w_0 + w_a(1-a)$$

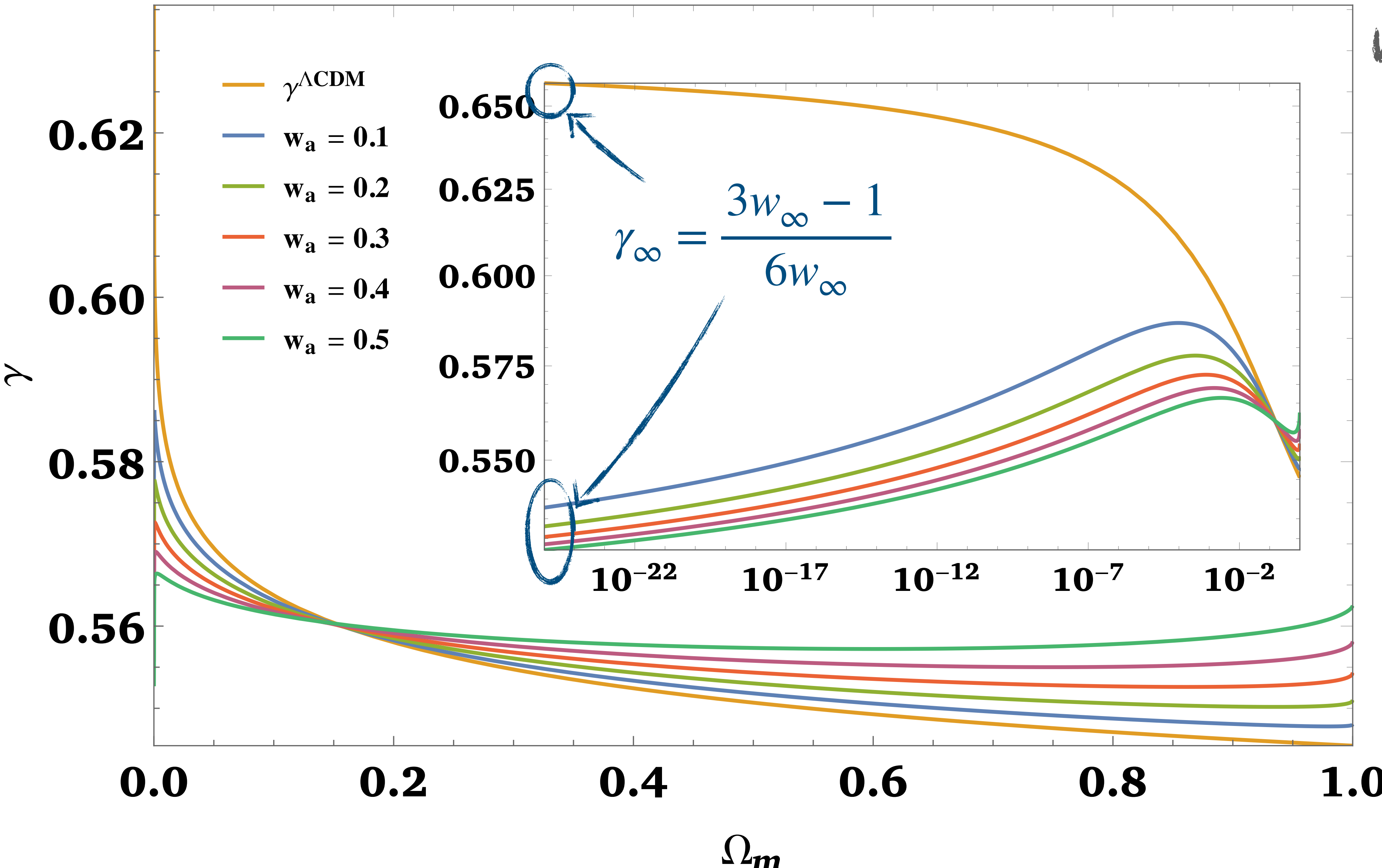
$$6w_{DE}(\Omega_m)\ln\Omega_m \frac{d\gamma}{d\ln\Omega_m} + 3w_{DE}(\Omega_m)\Omega_{DE}(2\gamma-1) + 1 + 2\Omega_m^{\gamma} - 3\,g\,\Omega_m^{1-\gamma} = 0$$

Using the useful relation:

We can solve for a!

$$w_{DE} = \frac{1}{3(1-\Omega_m)}\frac{d\ln\Omega_m}{d\ln a}$$

$$a(\Omega_m) \to w_{DE}(\Omega_m) \to \gamma(\Omega_m)$$



We restrict ourselves to

$$\boxed{w_{-\infty} = w_0 + w_a < 0}$$

To ensure MD

&

$$\boxed{w_a > 0}$$

To ensure DE Domination

Beyond General Relativity

MG enters through the source term in the equation

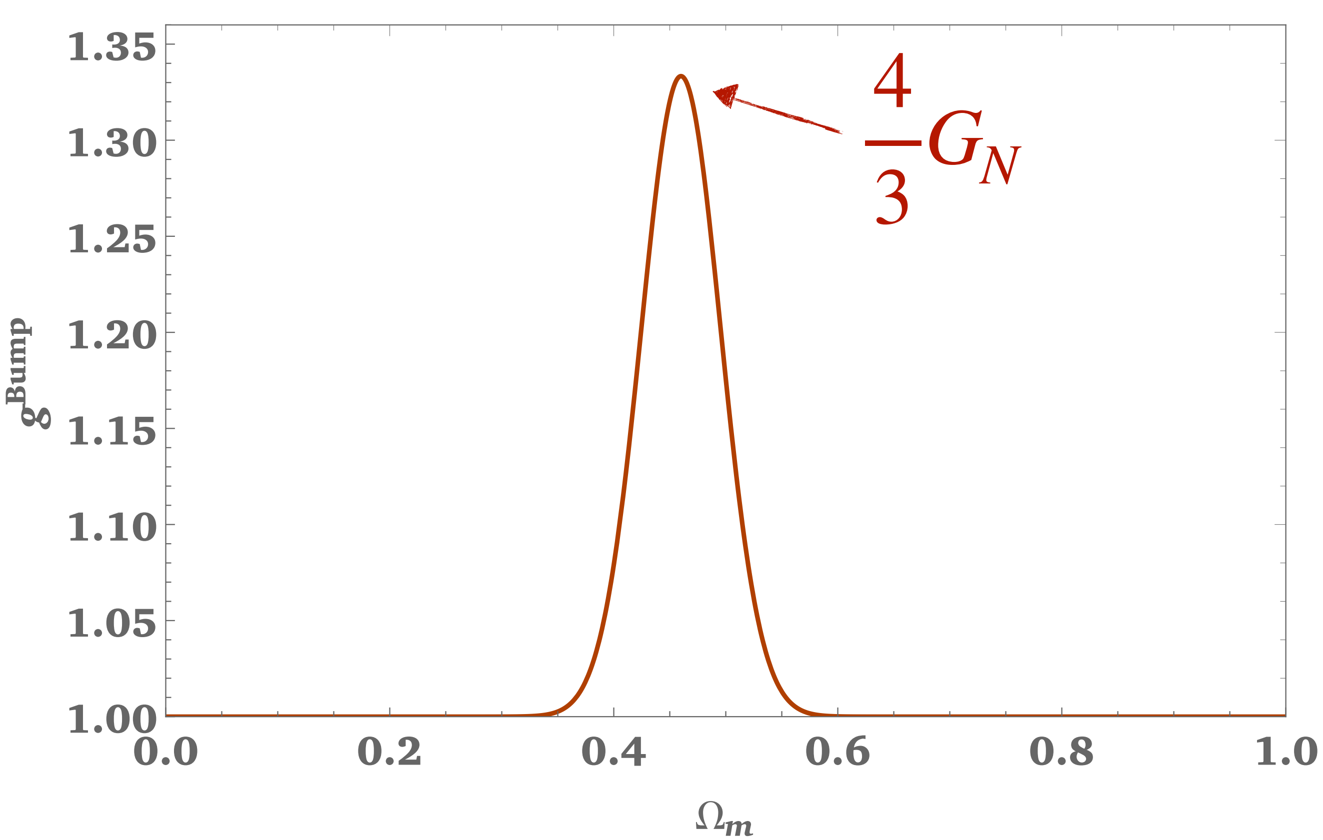
$$\frac{df}{d\ln a} + f^2 + \left(2 + \frac{\dot{H}}{H^2}\right) f = \frac{3}{2} g\, \Omega_m ,$$

Notice how γ is more sensitive to MG at early times! ($\Omega_m \sim 1$)

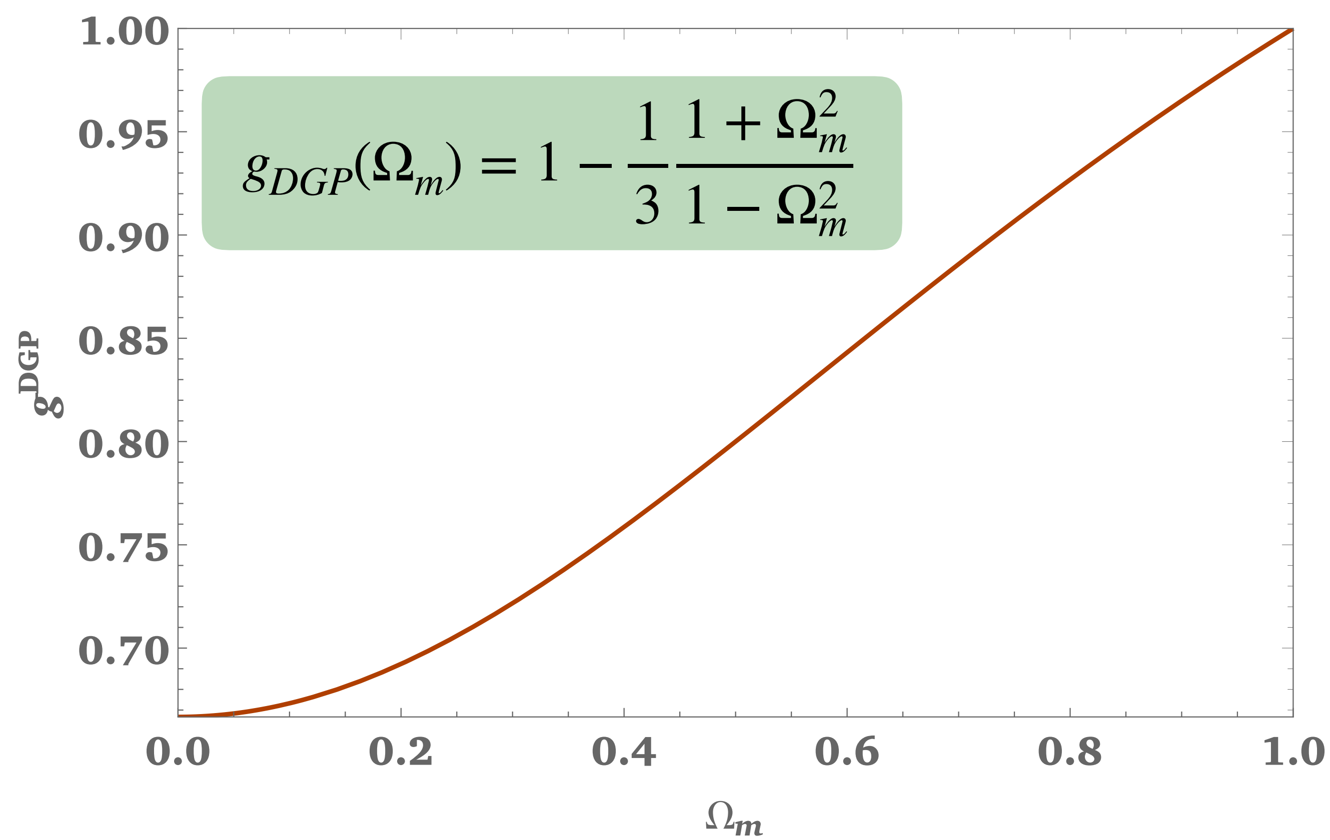
Modified Poisson equation $-k^2\,\Phi = 4\pi G_{\rm eff}\,\rho$

$$G \rightarrow G_{\rm eff}\,(a,k)$$

→ A Bump/Dip in $G_{\rm eff}$

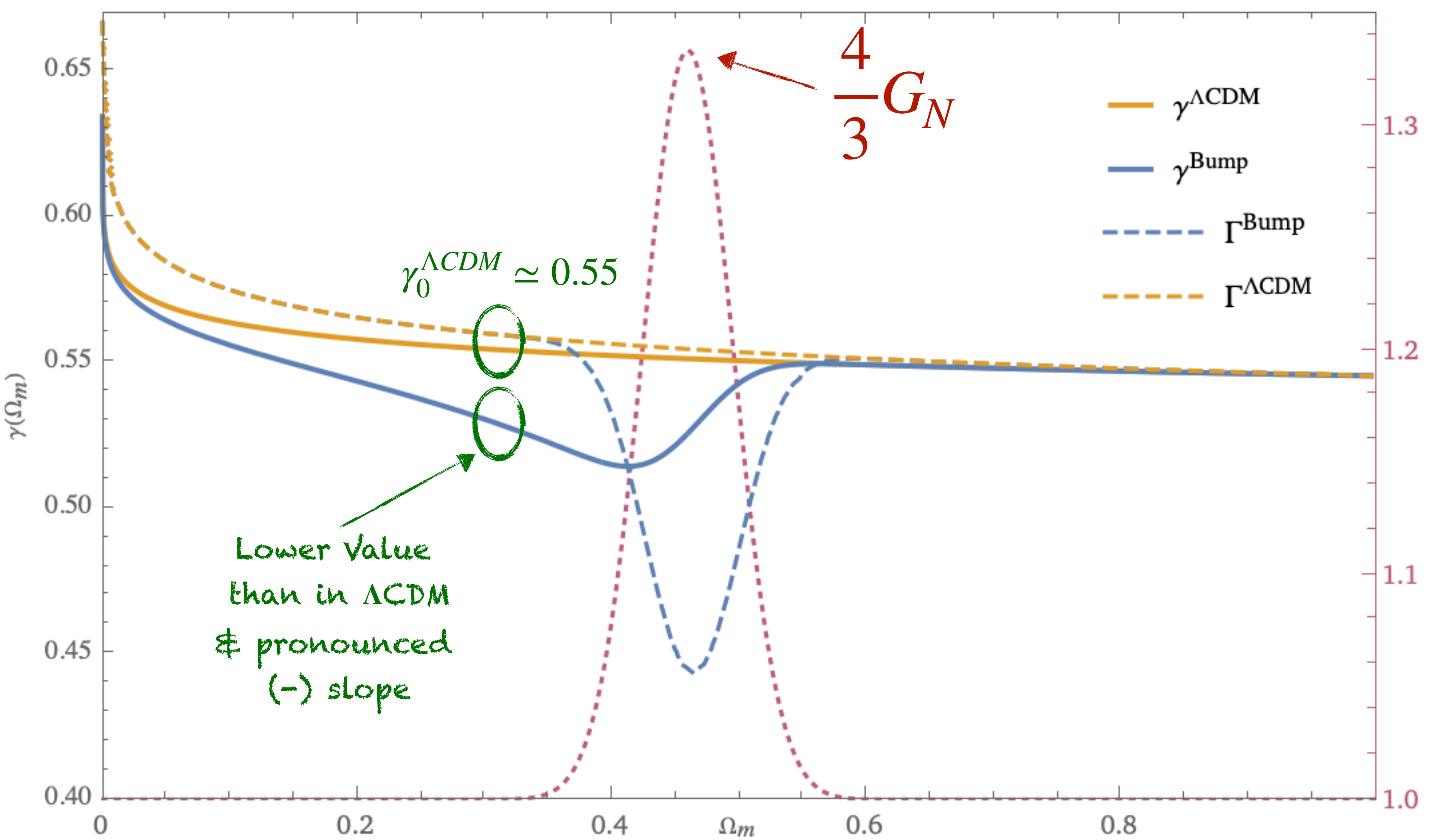


→ Braneworld Gravity (DGP)



Beyond GR: A bump in G_{eff}

→ Behaviour previously found in many f(R) theories...



$$\ddot{\delta}_m + 2H\dot{\delta}_m = 4\pi G_{\text{eff}}\rho_m\delta_m$$

As G_{eff} ↗, f also ↗

$$\gamma = \frac{\ln f}{\ln \Omega_m}, \quad \Omega_m \in [0,1]$$

→ γ decreases!

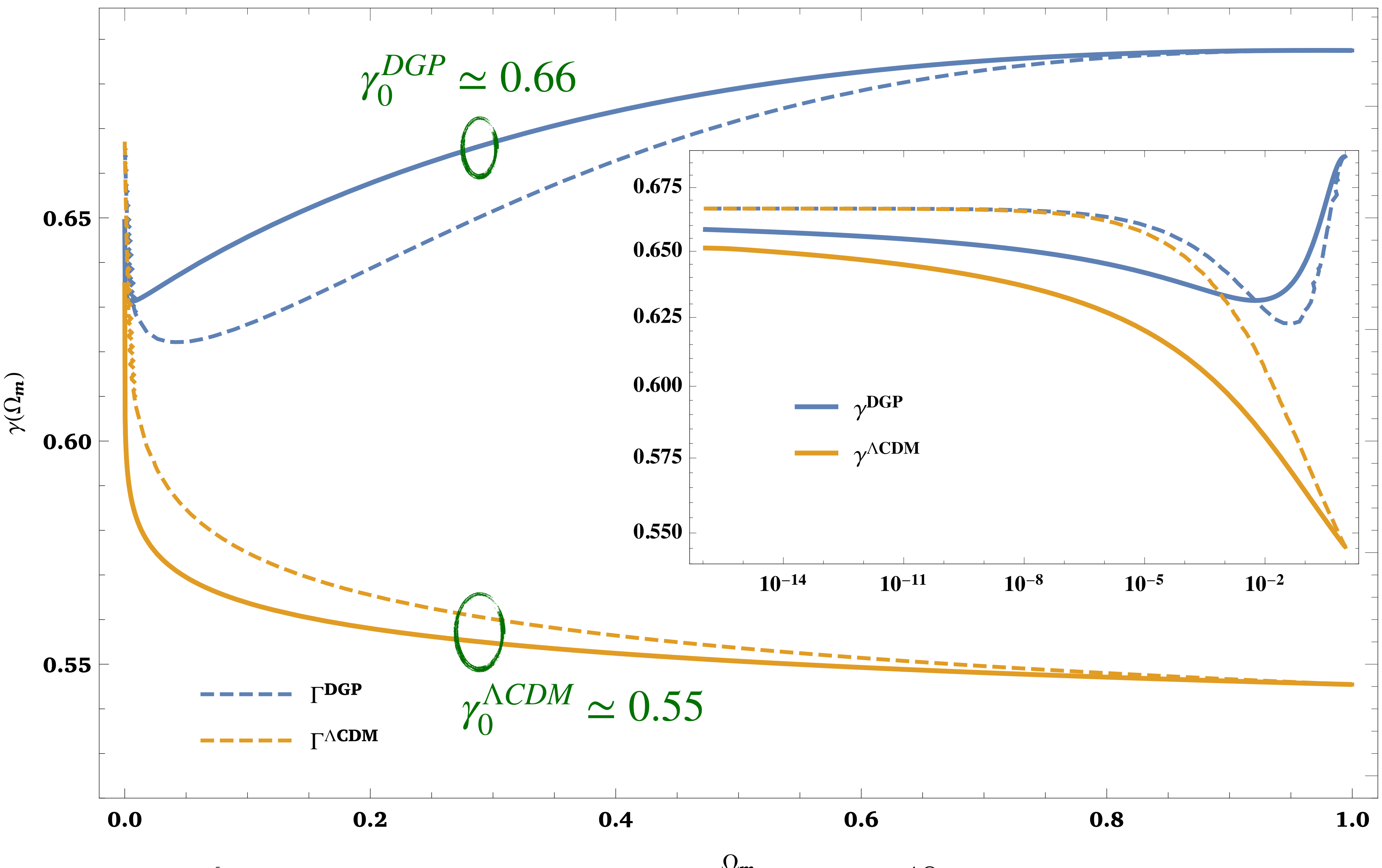
We could even infer our position with respect to the bump!

Beyond GR: DGP Models γ^{DGP}

Dvali-Gabadadze-Porrati : Braneworld Gravity

$$w_{DGP}(\Omega_m) = -(1+\Omega_m)^{-1}$$

$$g_{DGP}(\Omega_m) = 1 - \frac{1}{3}\frac{1+\Omega_m^2}{1-\Omega_m^2}$$



In these models:

$$g_{-\infty}^{DGP} = 1$$

$$w_{-\infty}^{DGP} = -\frac{1}{2}$$

$$\gamma_{-\infty}^{GR}(w = -\frac{1}{2}) = \frac{9}{16}$$

$$\gamma_{-\infty} = \frac{3(1 - w_{-\infty} - d)}{5 - 6w_{-\infty}}$$

$$\left(\frac{dg}{d\Omega_m}\right)_{-\infty} = \frac{1}{3} \neq 0$$

$$\gamma_{-\infty}^{DGP} = \frac{11}{16}$$

Conclusions

- A global analysis of the behaviour of γ starting deep in MD ($\Omega_m = 1$) up until the far future ($\Omega_m = 0$) yields some interesting properties

- Variable EoS $w_{DE}(a) = w_0 + w_a (1 - a)$

  ↳ Follows essentially the same phenomenology as $\gamma^{\Lambda CDM}$ on low-Z, with (very) different behaviours in the past & future

- Beyond GR: A Bump in $G_{\rm eff}$

  ↳ Yields Lower values for $\gamma_0 \equiv \gamma(z=0)$ than ΛCDM (+) Slope!

- Beyond GR: DGP Models

  ↳ Explicit origin of $\gamma_{-\infty}^{DGP} = \dfrac{11}{16}$ coming from $\left(\dfrac{d\,g}{d\Omega_m}\right)_{-\infty} = \dfrac{1}{3} \neq 0$

- An (accurate enough) estimation of γ & $\gamma'(z)$ could indicate a departure from ΛCDM (&/or GR)