Strongly non-geodesic motion in inflation

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Based on **1804.11279**, **1805.12563**, **1902.03221**, **1907.10403** with J. Fumagalli, L. Pinol, S. Renaux-Petel and J. Ronayne

Multi-field inflation and curved field space

The paradigm of **single-field inflation** is amazingly successful... but also unrealistic

More plausible is that, besides the **inflaton**, other fields were present

$$-\frac{1}{2}(\partial\phi)^2 - V(\phi) \quad \rightarrow \quad -\frac{1}{2}\,\delta_{IJ}\partial^\mu\phi^I\partial_\mu\phi^J - V(\phi^I)$$

- Well motivated theoretically (string theory and supergravity)
- Not problematic — if extra fields were heavy (compared to the Hubble scale H), dynamics can still be effectively single-field

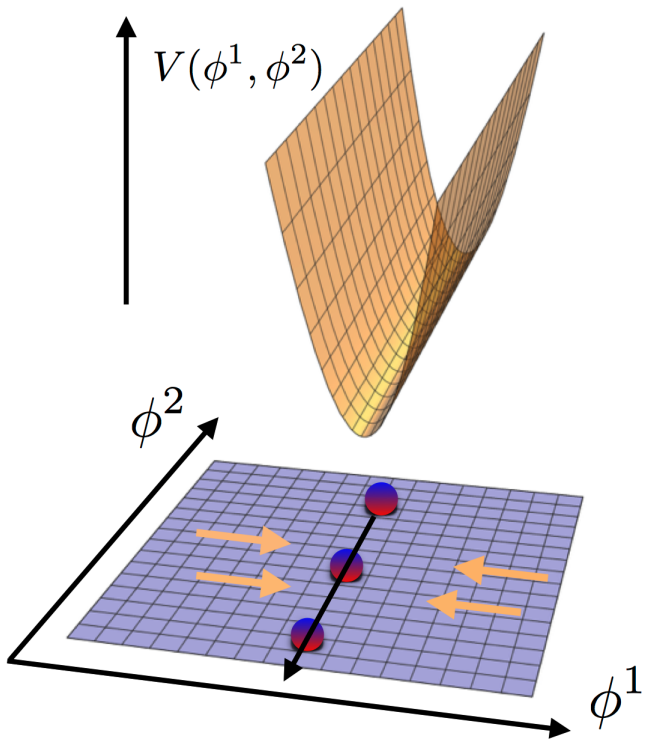


Image credit: S. Renaux-Petel

Multi-field inflation and curved field space

But this is still a bit simplistic...

- More generally, higher dimension operators will modify the kinetic structure of the theory

$$-\frac{1}{2}\,\delta_{IJ}\partial^\mu\phi^I\partial_\mu\phi^J \to -\frac{1}{2}\,G_{IJ}(\phi)\partial^\mu\phi^I\partial_\mu\phi^J$$

$\to$ **Curved internal field space**

$$G_{IJ}(\phi)d\phi^I d\phi^J = d(\phi^1)^2 + d(\phi^2)^2 + \frac{(\phi^1)^2}{M^2}\,d(\phi^2)^2 + \frac{(\phi^2)^2}{M^2}\,d\phi^1 d\phi^2 + \cdots$$

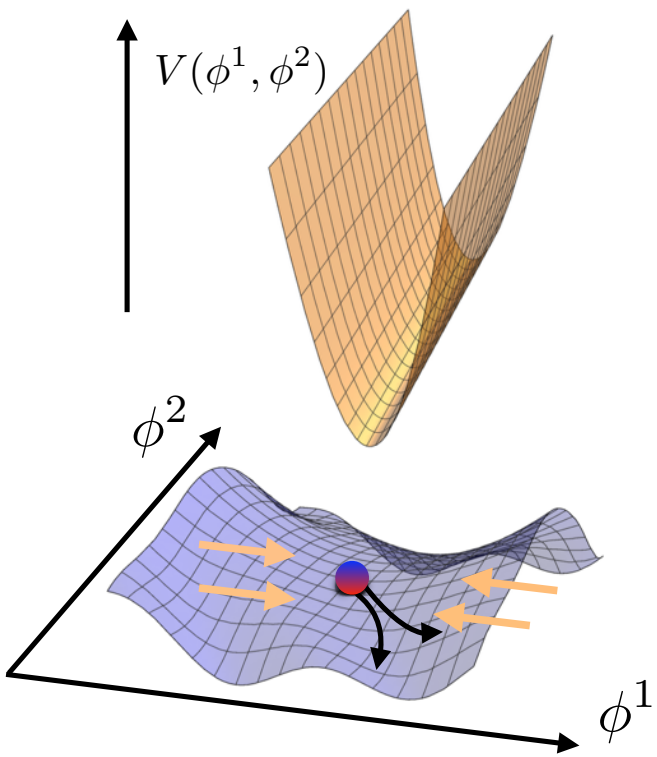


Image credit: S. Renaux-Petel

Multi-field inflation and curved field space

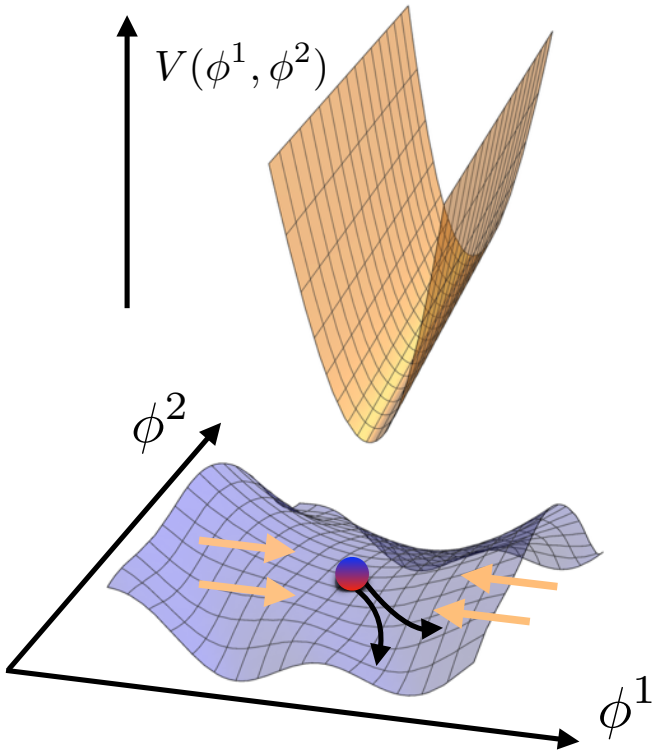
- The field space curvature is characterized by an energy scale M

$$\mathcal{R}_{IJKL} \sim \frac{1}{M^2}$$

The scale M need not be too large compared to Hubble

$$H < M < M_{\rm Pl}$$

so that the curvature of the field space may lead to sizable effects



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Multi-field inflation and curved field space

$$S = \int d^4x\sqrt{-g}\left[\frac{M_{\rm Pl}^2}{2}R(g) - \frac{1}{2}G_{IJ}(\phi)\partial^\mu\phi^I\partial_\mu\phi^J - V(\phi)\right]$$

Inflationary background

$$\bar{g}_{\mu\nu}dx^\mu dx^\nu = -dt^2 + a^2(t)d\vec{x}^2\,, \qquad \phi^I = \bar{\phi}^I(t)$$

Eqs. of motion

$$\mathcal{D}_t\dot{\bar{\phi}}^I + 3H\dot{\bar{\phi}}^I + G^{IJ}V_{,J} = 0$$

$$\dot{\sigma}^2 = 2M_{\rm Pl}^2H^2\epsilon\,, \qquad V = M_{\rm Pl}^2H^2(3-\epsilon)$$

with

$$\dot{\sigma} \equiv \sqrt{G_{IJ}\dot{\bar{\phi}}^I\dot{\bar{\phi}}^J}$$

$$\epsilon \equiv -\dot{H}/H^2$$

$$\mathcal{D}_tA^I \equiv \dot{A}^I + \Gamma^I_{JK}\dot{\bar{\phi}}^JA^K$$

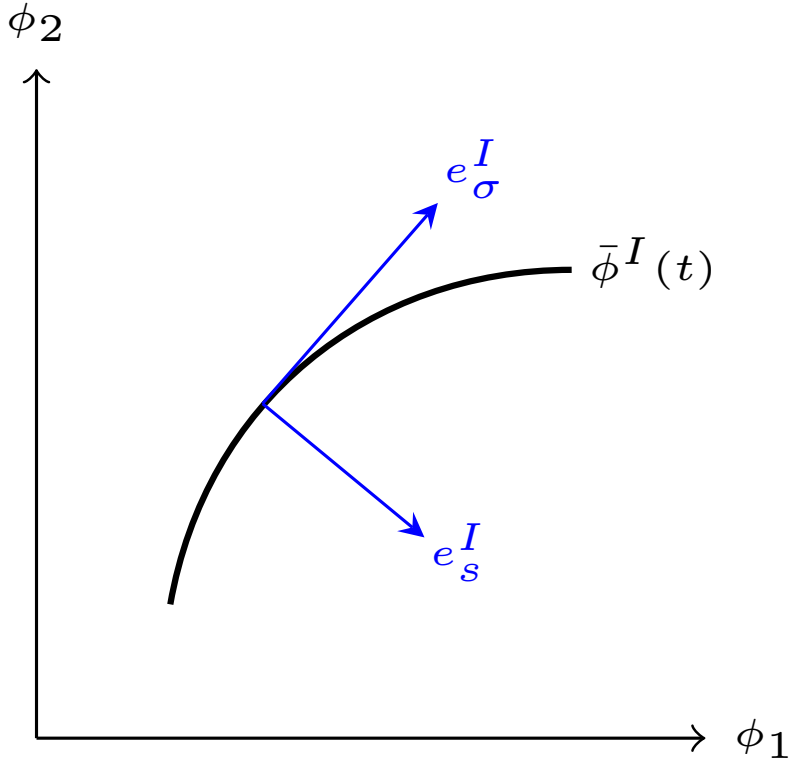
Strongly non-geodesic motion

It is useful to introduce an **adiabatic-entropic basis**

$$e^I_\sigma = \dot{\bar{\phi}}^I / \dot{\sigma}\,, \quad G_{IJ} e^I_\sigma e^J_s = 0$$

$e^I_\sigma \to$ adiabatic direction

$e^I_s \to$ entropic direction



Time derivative of basis vectors

$$\mathcal{D}_t e^I_\sigma = H \eta_\perp e^I_s\,, \qquad \mathcal{D}_t e^I_s = -H \eta_\perp e^I_\sigma$$

$\eta_\perp \to$ **bending parameter**

Groot Nibbelink & Van Tent (2002)

Strongly non-geodesic motion

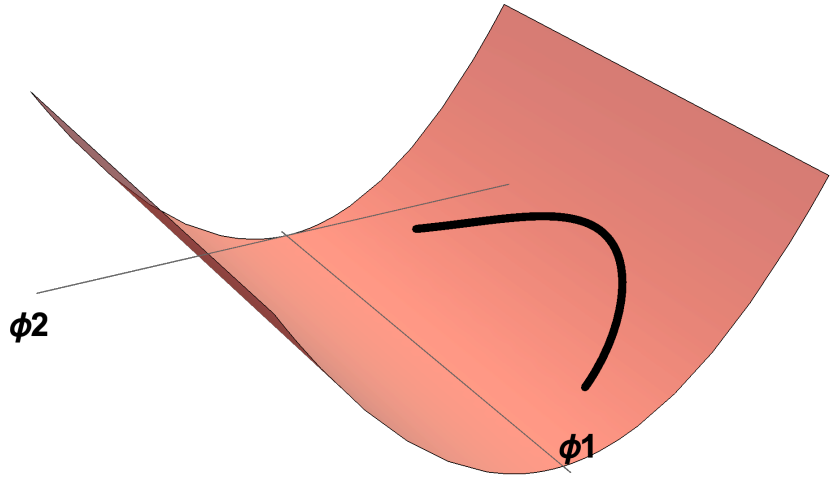
- $\eta_\perp = 0 \;\rightarrow\;$ geodesic motion (in field space)
- $|\eta_\perp| \gg 1 \;\rightarrow\;$ **strongly non-geodesic motion**

Projecting the eq. of motion along e^I_σ and e^I_s

$$\ddot{\sigma} + 3H\dot{\sigma} + V_{,\sigma} = 0\,, \qquad H\dot{\sigma}\eta_\perp + V_{,s} = 0$$

Remarks

- Inflation must occur on the slope of the potential to support $|\eta_\perp| \gg 1$
- This can naturally be achieved with curved field space



Strongly non-geodesic motion

- Examples of models
  - Hyperinflation Brown (2018)

$$G_{IJ}d\phi^I d\phi^J = d\rho^2 + M^2 \sinh^2(\rho/M) d\theta^2 \,, \qquad V = V(\rho)$$

  - Sidetracked inflation SGS, Renaux-Petel & Ronayne (2018)

$$G_{IJ}d\phi^I d\phi^J = \left(1 + \frac{2\chi^2}{M^2}\right) d\varphi^2 + d\chi^2 \,, \qquad V = V(\varphi) + \frac{m_h^2}{2}\chi^2$$

  - Angular inflation Christodoulidis et al. (2018)

$$G_{IJ}d\phi^I d\phi^J = \alpha \frac{d\varphi_1^2 + d\varphi_2^2}{(1 - \varphi_1^2 - \varphi_2^2)^2} \,, \qquad V = \frac{\alpha}{2}(m_1^2\varphi_1^2 + m_2^2\varphi_2^2)$$

- Unified descriptions
  - Dynamical attractors
    Bjorkmo (2019), Christodoulidis et al. (2019)
  - Effective field theory of perturbations
    SGS & Renaux-Petel (2018)

Strongly non-geodesic motion

Why is inflation with strongly non-geodesic motion interesting?

- Occurs naturally when field space has negative curvature
- Consistent with swampland conjectures Achucarro & Palma (2018)

$$\frac{M_{\rm Pl}|\nabla V|}{V} \gtrsim 1\,, \qquad |\nabla V| = \sqrt{G^{IJ}V_{,I}V_{,J}}$$

  - Single-field: $\epsilon \sim \left(\frac{M_{\rm Pl}|\nabla V|}{V}\right)^2 \ll 1$
  - Multi-field: $\epsilon \sim \frac{1}{1+\eta_\perp^2}\left(\frac{M_{\rm Pl}|\nabla V|}{V}\right)^2 \ll 1$

- Large non-Gaussianities with heavy fields

$$B_\zeta \sim \left(\frac{1}{c_s^2}-1\right)\mathcal{O}(1)+\mathcal{O}(\epsilon)\,, \qquad \frac{1}{c_s^2}-1 \sim \frac{\eta_\perp^2 H^2}{m_{\rm heavy}^2}$$

Linear perturbations

$$S = \int d^4x\sqrt{-g}\left[\frac{M_{\rm Pl}^2}{2}R(g) - \frac{1}{2}G_{IJ}(\phi)\partial^\mu\phi^I\partial_\mu\phi^J - V(\phi)\right]$$

In the flat gauge the metric is unperturbed and we define

$$\zeta = a^2\sqrt{2\epsilon}\, e^I_\sigma \delta\phi_I \quad \rightarrow \quad \text{curvature perturbation}$$

$$v_s = a\, e^I_s \delta\phi_I \quad \rightarrow \quad \text{entropic perturbation}$$

At quadratic order

$$S^{(2)} = \frac{1}{2}\int d\tau d^3x\Big[2a^2\epsilon\big(\zeta'^2 - k^2\zeta^2\big) + v_s'^2 - k^2 v_s^2 + \left(\frac{a''}{a} - m_s^2\right)a^2 v_s^2 - 4a^2\sqrt{2\epsilon}\, H\eta_\perp v_s\zeta'\Big]$$

Effective entropic mass

$$m_s^2 \equiv V_{;ss} + \epsilon R_{\rm fs} H^2 M_{\rm Pl}^2 - \eta_\perp^2 H^2$$

Linear perturbations

$$m_s^2 \equiv V_{;ss} + \epsilon R_{\rm fs} H^2 M_{\rm Pl}^2 - \eta_\perp^2 H^2$$

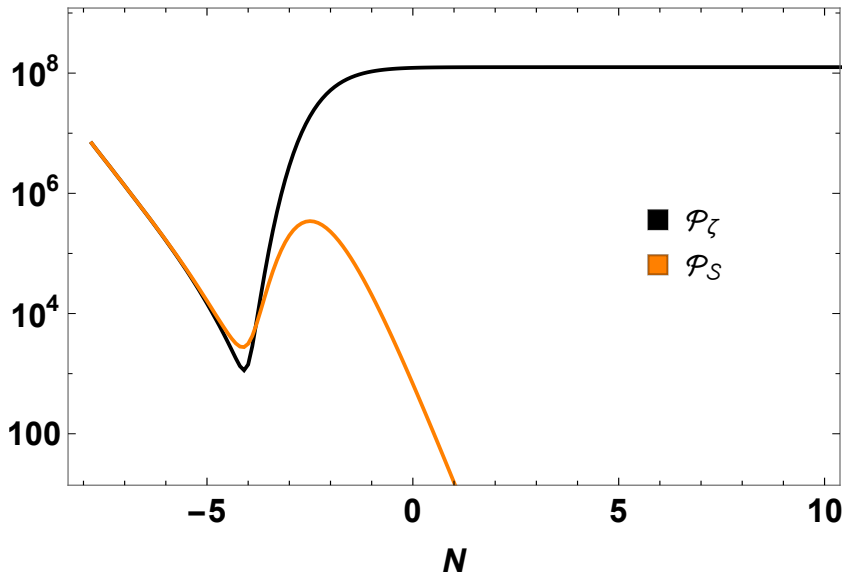
- Crucial observation is that we can have $m_s^2 < 0$

  $\rightarrow$ **tachyonic instability**

- Instability is typically only transient, since $\eta_\perp \rightarrow 0$ at the end of inflation

- But it leads to an exponential enhancement of the scalar power spectrum $\mathcal{P}_\zeta$

  Cremonini et al. (2011)



Linear perturbations

Eq. of motion for entropic mode

$$v_s'' + k^2 v_s + (m_s^2 - 2H^2) a^2 v_s = -2a^2 \sqrt{2\epsilon}\, H \eta_\perp \zeta'$$

Suppose $\quad |m_s^2| \gg k^2/a^2\,,\; H^2\,,\; \partial_\tau^2$

$$\rightarrow \qquad v_s \simeq -\frac{2\sqrt{2\epsilon}\, H \eta_\perp}{m_s^2}\, \zeta'$$

Substitute to get an **effective action** for ζ

$$S_{\rm eff}^{(2)} = \int d\tau d^3x\, a^2 \epsilon \left[\frac{\zeta'^2}{c_s^2} - k^2 \zeta^2 \right]$$

where c_s is the effective **speed of sound**

$$\frac{1}{c_s^2} \equiv 1 + \frac{4H^2 \eta_\perp^2}{m_s^2}$$

Inflation with imaginary speed of sound

$$S^{(2)}_{\rm eff} = \int d\tau d^3x\, a^2 \epsilon \left[\frac{\zeta'^2}{c_s^2} - k^2 \zeta^2 \right] , \qquad \frac{1}{c_s^2} \equiv 1 + \frac{4H^2\eta_\perp^2}{m_s^2}$$

In models with strongly non-geodesic motion

$$\eta_\perp^2 \gg 1 , \qquad m_s^2 \sim -H^2\eta_\perp^2 \qquad \rightarrow \qquad c_s^2 = -\mathcal{O}(1)$$

$\rightarrow$ **imaginary speed of sound**

But if $c_s^2 < 0$ then ζ is a ghost and gradient-unstable

- This makes sense — a tachyonic instability becomes a UV-sensitive instability in the EFT
- An EFT with a ghost? Wouldn't it be...
  - catastrophic? no – a Lorentz breaking EFT has a minimum timescale $1/\Lambda_{\rm cutoff}$ for any instability
  - useless? maybe – observables will be sensitive to $\Lambda_{\rm cutoff}$, EFT can't make quantitative predictions

Inflation with imaginary speed of sound

- We write the cutoff of the EFT in terms of a dimensionless parameter x

$$\frac{k|c_s|}{a} < xH$$

with $x \gg 1$

- In terms of time evolution, the EFT is valid for times τ such that

$$k|c_s|\tau + x > 0$$

with τ the conformal time, $\tau \in (-\infty, 0)$

- The curvature perturbation

$$\zeta_k(\tau) = \frac{\alpha_k}{k^{3/2}}\left(e^{k|c_s|\tau+x}(k|c_s|\tau - 1) - \rho_k e^{i\theta_k} e^{-k|c_s|\tau-x}(k|c_s|\tau + 1)\right)$$

has exponentially growing and decaying modes, as expected

Inflation with imaginary speed of sound

$$\zeta_k(\tau) = \frac{\alpha_k}{k^{3/2}}\left(e^{k|c_s|\tau+x}(k|c_s|\tau-1) - \rho_k e^{i\theta_k} e^{-k|c_s|\tau-x}(k|c_s|\tau+1)\right)$$

We parametrize the coefficients of the mode function in terms of α_k, ρ_k and θ_k (all real)

Remarks

- No Bunch–Davies vacuum — no minimum energy state
- Physically we expect $\rho_k = O(1)$, i.e. we expect the two modes to be excited in roughly the same way
- Quantization condition gives the constraint

$$\alpha_k^2 \rho_k \sin(\theta_k) = \frac{H^2}{4\epsilon|c_s|M_{\rm Pl}^2}$$

Inflation with imaginary speed of sound

Power spectrum

$$\mathcal{P}_\zeta(k) = \frac{\alpha_k^2}{2\pi^2}\left(e^{2x} + 2\rho_k \cos(\theta_k) + \rho_k^2 e^{-2x}\right)$$

- Very ugly result at first sight — depends on all the unknowns α_k, ρ_k, θ_k and x
- But recall that $x \gg 1$, and that we expect $\rho_k \sim 1$, $\alpha_k^2 \sim \frac{H^2}{4\epsilon|c_s|M_{\rm Pl}^2}$, so

$$\mathcal{P}_\zeta(k) \simeq \frac{\alpha_k^2}{2\pi^2}\, e^{2x} \sim \frac{H^2}{8\pi^2\epsilon|c_s|M_{\rm Pl}^2}\, e^{2x}$$

The tensor-to-scalar ratio will then be suppressed by e^{-2x} relative to single-field expectation

Inflation with imaginary speed of sound

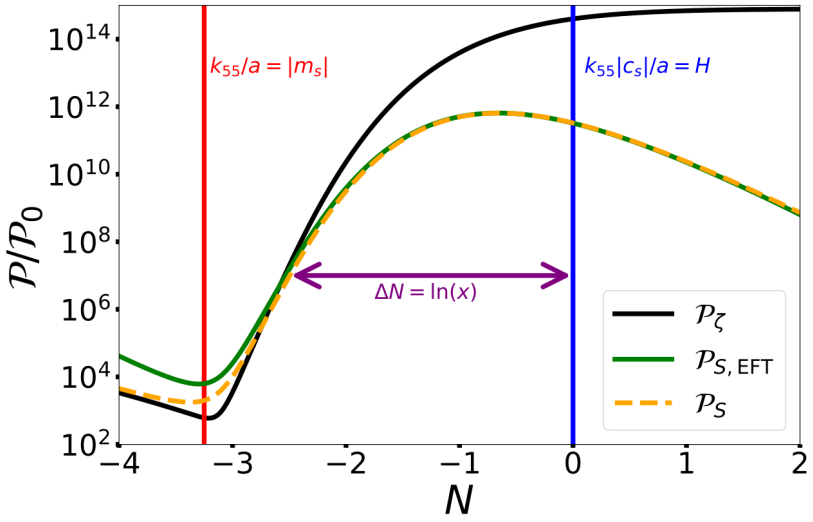
- Recall that x determines the regime of validity of the EFT

$$\tau \in \left(-\frac{x}{k|c_s|} \, , \, 0 \right)$$

- It is undetermined within the EFT, but we can estimate it via a "matching calculation"

- For instance, we can compare the numerical entropic power spectrum $\mathcal{P}_{\mathcal{S}}$ with the one derived from the relation

$$v_s \simeq -\frac{2\sqrt{2\epsilon}\, H \eta_\perp}{m_s^2}\, \zeta'$$



EFT of single-field inflation

- So far we've only discussed the power spectrum or 2-point function
- To derive higher-point correlations (**non-Gaussianities**) we have three options
  - Compute numerically in the full two-field theory
    so far only possible for the bispectrum
    e.g. Ronayne & Mulryne (2018)
  - Integrate out the entropic mode beyond quadratic order
    so far only possible for the bispectrum
    SGS, Pinol & Renaux-Petel (2019)
  - Use the complete EFT of single-field inflation
    Creminelli et al. (2006), Cheung et al. (2007)

EFT of single-field inflation

Inflation can be described in a model-independent way using effective field theory (EFT)

As with any EFT, we construct the action in three steps

- Identify the light degrees of freedom
  - Inflaton $\phi = \bar{\phi}(t) + \delta\phi(t, \vec{x})$
  - Graviton $g_{\mu\nu} = \bar{g}_{\mu\nu}^{\mathrm{FLRW}}(t) + h_{\mu\nu}(t, \vec{x})$
- Identify the relevant symmetries
  - Unbroken spatial diffeomorphisms $\vec{\xi}(t, \vec{x})$

    $\vec{x} \to \vec{x} + \vec{\xi}, \qquad \delta\phi \to \delta\phi$
  - Broken time diffeomorphisms $\xi^0(t, \vec{x})$

    $t \to t + \xi^0\,, \qquad \delta\phi \to \delta\phi + \dot{\bar{\phi}}(t)\xi^0$
- Write the most general action consistent with the symmetries, as a series in perturbations $\delta\phi, h_{\mu\nu}$ and in derivatives ∂_μ

EFT of single-field inflation

To start it's useful to choose a gauge where ξ^0 is such that $\delta\phi = 0$ (unitary gauge)

The most general action is

$$S = \int d^4x\sqrt{-g}\left[\frac{M_{\rm Pl}^2}{2}R + M_{\rm Pl}^2\dot{H}g^{00} - M_{\rm Pl}^2(3H^2+\dot{H}) + F(\delta g^{00}, \delta K_{\mu\nu}, \delta R_{\mu\nu\rho\sigma}; \nabla_\mu; t)\right]$$

Here F is an arbitrary function of

$\delta g^{00} = g^{00} + 1$

extrinsic curvature $\delta K_{\mu\nu}$ (uncontracted 0 indices allowed)

Riemann tensor $\delta R_{\mu\nu\rho\sigma}$ (uncontracted 0 indices allowed)

derivatives thereof

explicit functions of t

EFT of single-field inflation

At lowest order in perturbations and in derivatives

$$F = \frac{1}{2} M_2(t)^4 (\delta g^{00})^2 + \frac{1}{6} M_3(t)^4 (\delta g^{00})^3$$

For the purpose of computing observables it's convenient to reintroduce the scalar degree of freedom that was "eaten" by the metric

Rather than reintroduce $\delta\phi$, we can reintroduce the Goldstone $\pi(t, \vec{x})$ associated to the breaking of time translations

$$t \to t + \pi \,, \qquad g^{00} \to \partial_\mu (t + \pi) \partial_\nu (t + \pi) g^{\mu\nu}$$

The final result simplifies in the **decoupling limit** where the mixing with gravity can be neglected

$$g^{00} \to -2\dot{\pi} - \dot{\pi}^2 + \frac{(\vec{\nabla}\pi)^2}{a^2}$$

EFT of single-field inflation

After this approximation our final action up to cubic order is

$$S = \int dt d^3x \, a^3 M_{\rm Pl}^2 \epsilon H^2 \left[\frac{\dot{\pi}^2}{c_s^2} - \frac{(\vec{\nabla}\pi)^2}{a^2} - \left(\frac{1}{c_s^2} - 1 \right) \left(\frac{\dot{\pi}(\vec{\nabla}\pi)^2}{a^2} + \frac{A}{c_s^2} \dot{\pi}^3 \right) \right]$$

The curvature perturbation is then $\zeta \simeq -H\pi$

- There are two relevant coefficients
  - Speed of sound: $\frac{1}{c_s^2} \equiv 1 + \frac{2M_2^2}{M_{\rm Pl}^2 \epsilon H^2}$
  - Coupling constant: $A \equiv -c_s^2 \left(1 - \frac{2}{3} \left(\frac{M_3}{M_2} \right)^4 \right)$
- They are undetermined within the EFT — to know them we need a UV completion

EFT of single-field inflation

The UV completion is our two-field model in curved field space, and the EFT is obtained upon integrating out the entropic perturbation v_s

- At quadratic we already derived the speed of sound

$$\frac{1}{c_s^2} = 1 + \frac{4H^2\eta_\perp^2}{m_s^2}$$

- To find A one needs to integrate out v_s at cubic order

$$A = -\frac{1-c_s^2}{2} + \frac{2\epsilon H^2 M_{\rm Pl}^2 R_{\rm fs}(1+2c_s^2)}{3m_s^2} - \frac{\sqrt{2\epsilon}\, M_{\rm Pl}(1-c_s^2)}{6\eta_\perp m_s^2}\left(V_{;sss} + \epsilon H^2 M_{\rm Pl}^2 R_{{\rm fs};s}\right)$$

SGS, Pinol & Renaux-Petel (2019)

Bispectrum with $c_s^2 < 0$

Cubic action

$$S_{\rm eff}^{(3)} = -\int d\tau d^3x \, \frac{a\epsilon M_{\rm Pl}^2}{H} \left(\frac{1}{|c_s|^2} + 1 \right) \left(\zeta' (\vec{\nabla}\zeta)^2 - \frac{A}{|c_s|^2} \zeta'^3 \right)$$

The **reduced bispectrum** is

$$\begin{aligned} f_{NL} &= \frac{B_\zeta(k_1, k_2, k_3)}{\mathcal{P}_\zeta(k_1)\mathcal{P}_\zeta(k_2) + \mathcal{P}_\zeta(k_2)\mathcal{P}_\zeta(k_3) + \mathcal{P}_\zeta(k_3)\mathcal{P}_\zeta(k_1)} \\ &= \frac{S(k_1, k_2, k_3)}{k_1^2/(k_2 k_3) + (2 \text{ perm.})} \end{aligned}$$

We write the **shape function** as

$$S(k_1, k_2, k_3) = S_{\zeta'(\vec{\nabla}\zeta)^2} + A \, S_{\zeta'^3}$$

with $A = \mathcal{O}(1)$

Bispectrum with $c_s^2 < 0$

$$\zeta_k(\tau) = \frac{\alpha_k}{k^{3/2}}\Big(e^{k|c_s|\tau+x}(k|c_s|\tau-1) - \rho_k e^{i\theta_k} e^{-k|c_s|\tau-x}(k|c_s|\tau+1)\Big)$$

Even though B_ζ and $\mathcal{P}_\zeta$ are UV sensitive

$$\mathcal{P}_\zeta \propto e^{2x}\,, \qquad B_\zeta \propto e^{4x}$$

the quantity f_{NL} is **UV insensitive** (with caveats)

The results again depend on the unknown parameters ρ_k, θ_k. But because $x \gg 1$ the outcome turns out to be "universal"

$$S_{\zeta'^3} = \frac{3}{4}\left(\frac{1}{|c_s|^2}+1\right)\Bigg\{-\frac{k_1k_2k_3}{(k_1+k_2+k_3)^3} + \frac{k_1k_2k_3}{\tilde{k}_1^3}\left[1 - e^{-x\tilde{k}_1/k_{\max}}\left(1 + x\frac{\tilde{k}_1}{k_{\max}} + \frac{x^2}{2}\frac{\tilde{k}_1^2}{k_{\max}^2}\right)\right]\Bigg\} + (2 \text{ perm.})$$

$\tilde{k}_1 \equiv k_2 + k_3 - k_1$, etc.

Bispectrum with $c_s^2 < 0$

$$S_{\zeta'^3} = \frac{3}{4}\left(\frac{1}{|c_s|^2}+1\right)\left\{-\frac{k_1k_2k_3}{(k_1+k_2+k_3)^3} + \frac{k_1k_2k_3}{\tilde{k}_1^3}\left[1-e^{-x\tilde{k}_1/k_{\max}}\left(1+x\frac{\tilde{k}_1}{k_{\max}}+\frac{x^2}{2}\frac{\tilde{k}_1^2}{k_{\max}^2}\right)\right]\right\} + (2 \text{ perm.})$$

$\tilde{k}_1 \equiv k_2 + k_3 - k_1$, etc.

Remarks

- Result is independent of α_k, ρ_k, θ_k
- Whenever $\tilde{k}_i > 0$, result is also independent of cutoff parameter x

  Equilateral shape

$$S_{\zeta'^3}(k,k,k) \simeq \frac{13}{6|c_s|^2}$$

Bispectrum with $c_s^2 < 0$

$$S_{\zeta'^3} = \frac{3}{4}\left(\frac{1}{|c_s|^2}+1\right)\left\{-\frac{k_1 k_2 k_3}{(k_1+k_2+k_3)^3} + \frac{k_1 k_2 k_3}{\tilde{k}_1{}^3}\left[1 - e^{-x\tilde{k}_1/k_{\max}}\left(1 + x\frac{\tilde{k}_1}{k_{\max}} + \frac{x^2}{2}\frac{\tilde{k}_1{}^2}{k_{\max}^2}\right)\right]\right\} + (2 \text{ perm.})$$

$\tilde{k}_1 \equiv k_2 + k_3 - k_1$, etc.

- On the triangle edges $\tilde{k}_i \to 0$. The result is still finite but with a power-law cutoff dependence

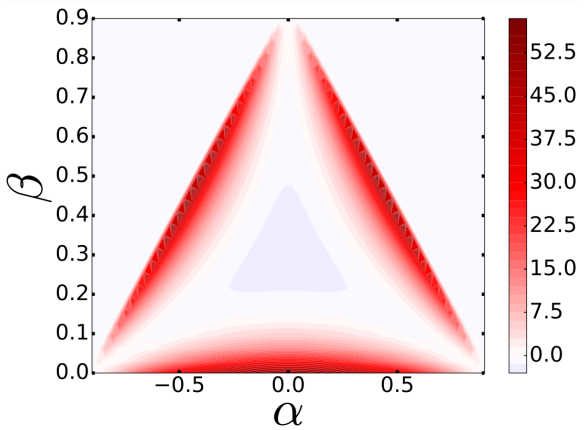
  Squashed configuration

  $$S_{\zeta'^3}(k, k/2, k/2) \simeq \frac{1}{128|c_s|^2}\left(39 + 4x^3\right)$$

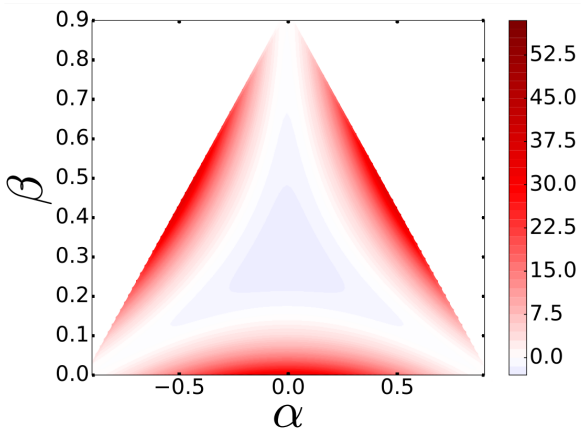
- On the triangle edges the non-Gaussianities are large — but not exponentially so
- The EFT predicts non-Gaussianities peaked on squashed shapes

Bispectrum with $c_s^2 < 0$

Test of the EFT with hyperinflation model

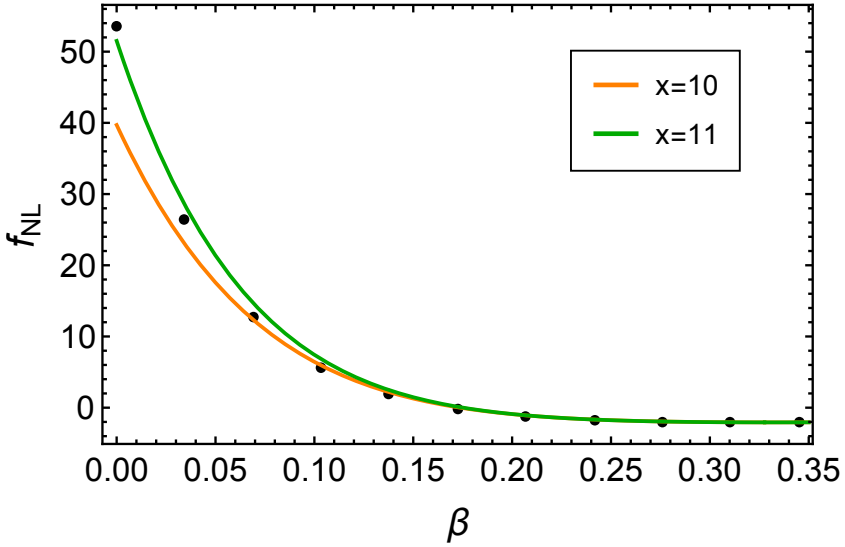


Numerical result in full theory



Analytical result in EFT

(using $x = 10$, $c_s^2 = -1$, $A = 1/3$)



Higher-point correlations with $c_s^2 < 0$

Higher-point correlation functions

- Just like for the bispectrum, the reduced n-point function is not exponentially amplified

$$\langle \zeta^n \rangle \sim e^{2x(n-1)}\,, \quad \langle \zeta^2 \rangle \sim e^{2x} \qquad \rightarrow \qquad \frac{\langle \zeta^n \rangle}{\langle \zeta^2 \rangle^{n-1}} \sim \text{“order 1”}$$

Bjorkmo, Ferreira & Marsh (2019)

- However, flattened shapes generically have a power-law enhancement

$$\frac{\langle \zeta^n \rangle}{\langle \zeta^2 \rangle^{n-1}} \sim \left[\left(\frac{1}{|c_s|^2} + 1 \right) x^3 \right]^{n-2}$$

Fumagalli, SGS, Pinol, Renaux-Petel & Ronayne (2019)

- These models are still under perturbative control, but strongly constrained by experimental bounds on $\langle \zeta^3 \rangle$ and $\langle \zeta^4 \rangle$

Summary

- Inflation with strongly non-geodesic motion
  - Well motivated by microscopic considerations
    many scalars, curved field space, swampland conjectures
  - Interesting observational signatures
    suppressed tensor-to-scalar ratio, large non-Gaussianities of flattened shapes
- Description from effective field theory
  - Unusual theory with a ghost and gradient instability, yet with a clear regime of validity
  - Results match very well first-principles numerical calculations
  - Allows to go beyond current numerical techniques

Thank you