Cosmological perturbations during the kinetic inflation in the Horndeski theory

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Plan of the talk

- Scalar fields in gravitational physics
- Horndeski model
- Introduction: Cosmological models with nonminimal derivative coupling
  - No potential
  - Cosmological constant
  - Power-law potential
  - The screening Horndeski cosmologies
- Perturbations
- Summary

Scalar fields in gravitational physics:

- gravitational potential in Newtonian gravity
- variation of "fundamental" constants
- Brans-Dicke theory initially elaborated to solve the Mach problem
- various compactification schemes
- the low-energy limit of the superstring theory
- scalar field as inflaton
- scalar field as dark energy and/or dark matter
- fundamental Higgs bosons, neutrinos, axions, ...
- etc...

In 1974, Horndeski derived the action of the most general scalar-tensor theories with second-order equations of motion
[G.Horndeski, *Second-Order Scalar-Tensor Field Equations in a Four-Dimensional Space*, IJTP **10**, 363 (1974)]

Horndeski Lagrangian:

$$L_{\mathrm{H}} = \sqrt{-g}\,(\mathcal{L}_2 + \mathcal{L}_3 + \mathcal{L}_4 + \mathcal{L}_5)$$

$$\begin{aligned}
\mathcal{L}_2 &= G_2(X,\Phi)\,,\\
\mathcal{L}_3 &= G_3(X,\Phi)\,\Box\Phi\,,\\
\mathcal{L}_4 &= G_4(X,\Phi)\,R + \partial_X G_4(X,\Phi)\,\delta^{\mu\nu}_{\alpha\beta}\,\nabla^\alpha_\mu\Phi\nabla^\beta_\nu\Phi\,,\\
\mathcal{L}_5 &= G_5(X,\Phi)\,G_{\mu\nu}\nabla^{\mu\nu}\Phi - \tfrac{1}{6}\,\partial_X G_5(X,\Phi)\,\delta^{\mu\nu\rho}_{\alpha\beta\gamma}\,\nabla^\alpha_\mu\Phi\nabla^\beta_\nu\Phi\nabla^\gamma_\rho\Phi\,,
\end{aligned}$$

where $X = -\frac{1}{2}(\nabla\phi)^2$, and $G_k(X,\Phi)$ are arbitrary functions,
and $\delta^{\lambda\rho}_{\nu\alpha} = 2!\,\delta^\lambda_{[\nu}\delta^\rho_{\alpha]}$, $\delta^{\lambda\rho\sigma}_{\nu\alpha\beta} = 3!\,\delta^\lambda_{[\nu}\delta^\rho_\alpha\delta^\sigma_{\beta]}$

Fab Four subclass of the Horndeski theory

There is a special subclass of the theory, sometimes called Fab Four (F4), for which the coefficients are chosen such that the Lagrangian becomes

$$L_{\rm F4} = \sqrt{-g}\,(\mathcal{L}_J + \mathcal{L}_P + \mathcal{L}_G + \mathcal{L}_R - 2\Lambda)$$

with

$$\begin{aligned}
\mathcal{L}_J &= V_J(\Phi)\, G_{\mu\nu} \nabla^\mu \Phi \nabla^\nu \Phi\,, \\
\mathcal{L}_P &= V_P(\Phi)\, P_{\mu\nu\rho\sigma} \nabla^\mu \Phi \nabla^\rho \Phi \nabla^{\nu\sigma} \Phi\,, \\
\mathcal{L}_G &= V_G(\Phi)\, R\,, \\
\mathcal{L}_R &= V_R(\Phi)\, (R_{\mu\nu\alpha\beta} R^{\mu\nu\alpha\beta} - 4 R_{\mu\nu} R^{\mu\nu} + R^2).
\end{aligned}$$

Here the double dual of the Riemann tensor is

$$P^{\mu\nu}{}_{\alpha\beta} = -\frac{1}{4}\, \delta^{\mu\nu\gamma\delta}_{\sigma\lambda\alpha\beta}\, R^{\sigma\lambda}{}_{\gamma\delta} = -R^{\mu\nu}{}_{\alpha\beta} + 2 R^\mu_{[\alpha} \delta^\nu_{\beta]} - 2 R^\nu_{[\alpha} \delta^\mu_{\beta]} - R \delta^\mu_{[\alpha} \delta^\nu_{\beta]}\,,$$

whose contraction is the Einstein tensor, $P^{\mu\alpha}{}_{\nu\alpha} = G^\mu{}_\nu$.

Fab Four subclass of the Horndeski theory

Fab Four Lagrangian:

$$L_{\mathrm{F4}} = \sqrt{-g}\,(\mathcal{L}_J + \mathcal{L}_P + \mathcal{L}_G + \mathcal{L}_R - 2\Lambda)$$

- The Fab Four model is distinguished by the *screening property* – it is the most general subclass of the Horndeski theory in which flat space is a solution, despite the presence of the cosmological term Λ.
- This property suggests that Λ is actually irrelevant and hence there is no need to explain its value.
- Indeed, however large Λ is, Minkowski space is always a solution and so one may hope that a slowly accelerating universe will be a solution as well.

Theory with nonminimal kinetic coupling

Action:

$$S=\frac{1}{2}\int d^4x\sqrt{-g}\left[M_{\rm Pl}^2 R-(\varepsilon\, g_{\mu\nu}+\eta\, G_{\mu\nu})\nabla^\mu\phi\nabla^\nu\phi-2V(\phi)\right]+S_{\rm m}$$

Field equations:

$$M_{\rm Pl}^2 G_{\mu\nu}=T^{(\phi)}_{\mu\nu}+\eta\,\Theta_{\mu\nu}+T^{(\rm m)}_{\mu\nu}$$

$$[\epsilon g^{\mu\nu}+\eta G^{\mu\nu}]\nabla_\mu\nabla_\nu\phi=V'_\phi$$

$$\begin{aligned}
T^{(\phi)}_{\mu\nu}&=\epsilon\left[\nabla_\mu\phi\nabla_\nu\phi-\tfrac{1}{2}g_{\mu\nu}(\nabla\phi)^2\right]-g_{\mu\nu}V(\phi),\\
\Theta_{\mu\nu}&=-\tfrac{1}{2}\nabla_\mu\phi\,\nabla_\nu\phi\,R+2\nabla_\alpha\phi\,\nabla_{(\mu}\phi R^\alpha_{\nu)}-\tfrac{1}{2}(\nabla\phi)^2G_{\mu\nu}+\nabla^\alpha\phi\,\nabla^\beta\phi\,R_{\mu\alpha\nu\beta}\\
&\quad+\nabla_\mu\nabla^\alpha\phi\,\nabla_\nu\nabla_\alpha\phi-\nabla_\mu\nabla_\nu\phi\,\Box\phi+g_{\mu\nu}\Big[-\tfrac{1}{2}\nabla^\alpha\nabla^\beta\phi\,\nabla_\alpha\nabla_\beta\phi+\tfrac{1}{2}(\Box\phi)^2\\
&\quad-\nabla_\alpha\phi\,\nabla_\beta\phi\,R^{\alpha\beta}\Big]\\
T^{(\rm m)}_{\mu\nu}&=(\rho+p)U_\mu U_\mu+pg_{\mu\nu}\,,
\end{aligned}$$

Notice: *The field equations are of second order!*

Cosmological models: General formulas

Based on 0910.0980, 1002.3478, 1204.6372, 1306.5090, 1510.03264, 1604.06085, 1703.04966, 1803.01429

Collaborators: Saridakis, Toporensky, Skugoreva, Matsumoto, Capozziello, Volkov, Starobinsky

Ansatz:

$$ds^2 = -dt^2 + a^2(t)d\mathbf{x}^2,$$

$$\phi = \phi(t)$$

$a(t)$ *cosmological factor*, $H = \dot{a}/a$ *Hubble parameter*

Field equations:

$$3M_{\rm Pl}^2 H^2 = \frac{1}{2}\dot{\phi}^2\left(\epsilon - 9\eta H^2\right) + V(\phi),$$

$$M_{\rm Pl}^2(2\dot{H} + 3H^2) = -\frac{1}{2}\dot{\phi}^2\left[\epsilon + \eta\left(2\dot{H} + 3H^2 + 4H\ddot{\phi}\dot{\phi}^{-1}\right)\right] + V(\phi),$$

$$\frac{d}{dt}\left[(\epsilon - 3\eta H^2)a^3\dot{\phi}\right] = -a^3\frac{dV(\phi)}{d\phi}$$

Cosmological models: I. No potential $V(\phi) \equiv 0$

Trivial model without kinetic coupling, i.e. $\eta = 0$

$$S = \frac{1}{2}\int d^4x\sqrt{-g}\left[M_{\rm Pl}^2 R - (\nabla\phi)^2\right]$$

Trivial model without kinetic coupling, i.e. $\eta = 0$

$$S = \frac{1}{2} \int d^4x \sqrt{-g} \left[M_{\rm Pl}^2 R - (\nabla\phi)^2 \right]$$

Solution:

$$a_0(t) = t^{1/3}; \quad \phi_0(t) = \frac{1}{2\sqrt{3\pi}} \ln t$$

$$ds_0^2 = -dt^2 + t^{2/3} d\mathbf{x}^2$$

$t = 0$ *is an initial singularity*

Model without free kinetic term, i.e. $\epsilon = 0$

$$S = \frac{1}{2}\int d^4x\sqrt{-g}\left[M_{\rm Pl}^2 R - \eta G^{\mu\nu}\phi_{,\mu}\phi_{,\nu}\right]$$

Model without free kinetic term, i.e. $\epsilon = 0$

$$S = \frac{1}{2} \int d^4x \sqrt{-g} \left[M_{\rm Pl}^2 R - \eta G^{\mu\nu} \phi_{,\mu} \phi_{,\nu} \right]$$

Solution:

$$a(t) = t^{2/3}; \quad \phi(t) = \frac{t}{2\sqrt{3\pi|\eta|}}, \quad \eta < 0$$

$$ds_0^2 = -dt^2 + t^{4/3} d\mathbf{x}^2$$

$t = 0$ *is an initial singularity*

Model for an ordinary scalar field ($\epsilon = 1$) with nonminimal kinetic coupling $\eta \neq 0$

$$S = \frac{1}{2} \int d^4x \sqrt{-g} \left[M_{\rm Pl}^2 R - (g^{\mu\nu} + \eta G^{\mu\nu}) \phi_{,\mu} \phi_{,\nu} \right]$$

Model for an ordinary scalar field ($\epsilon = 1$) with nonminimal kinetic coupling $\eta \neq 0$

$$S = \frac{1}{2}\int d^4x\sqrt{-g}\left[M_{\rm Pl}^2 R - (g^{\mu\nu} + \eta G^{\mu\nu})\phi_{,\mu}\phi_{,\nu}\right]$$

Asymptotic for $t \to \infty$:

$$a(t) \sim a_0(t) = t^{1/3}; \quad \phi(t) \sim \phi_0(t) = \frac{1}{2\sqrt{3\pi}}\ln t$$

Notice: *At large times the model with $\eta \neq 0$ has the same behavior like that with $\eta = 0$*

Asymptotics for early times

The case $\eta < 0$:

$$a_{t\to 0} \approx t^{2/3}; \quad \phi_{t\to 0} \approx \frac{t}{2\sqrt{3\pi|\eta|}}$$

$$ds^2_{t\to 0} = -dt^2 + t^{4/3} d\mathbf{x}^2$$

$t = 0$ *is an initial singularity*

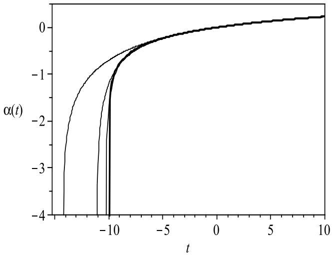
The case $\eta > 0$:

$$a_{t\to -\infty} \approx e^{H_\eta t}; \quad \phi_{t\to -\infty} \approx Ce^{-t/\sqrt{\eta}}$$

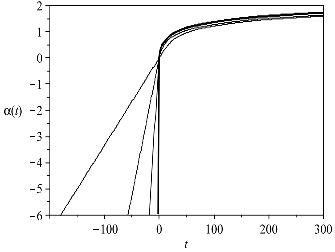
$$ds^2_{t\to -\infty} = -dt^2 + e^{2H_\eta t} d\mathbf{x}^2$$

de Sitter asymptotic with $H_\eta = 1/\sqrt{9\eta}$

Plots of $\alpha = \ln a$ in case $\eta \neq 0$, $\epsilon = 1$, $V = 0$.



(a) $\eta < 0$;
$\eta = 0; -1; -10; -100$



(b) $\eta > 0$;
$\eta = 0; 1; 10; 100$

De Sitter asymptotics: $\alpha(t) = \dfrac{t}{\sqrt{9\eta}} \quad \Rightarrow \quad H = \dfrac{1}{\sqrt{9\eta}}$

Notice: *In the model with nonmnimal kinetic coupling one get de Sitter phase without any potential!*

Models with the constant potential $V(\phi) = M_{\rm Pl}^2\Lambda = const$

$$S = \int d^4x\sqrt{-g}\left[M_{\rm Pl}^2(R - 2\Lambda) - [\epsilon g^{\mu\nu} + \eta G^{\mu\nu}]\phi_{,\mu}\phi_{,\nu}\right]$$

Models with the constant potential $V(\phi) = M_{\rm Pl}^2 \Lambda = const$

$$S = \int d^4x\sqrt{-g}\left[M_{\rm Pl}^2(R - 2\Lambda) - [\epsilon g^{\mu\nu} + \eta G^{\mu\nu}]\phi_{,\mu}\phi_{,\nu}\right]$$

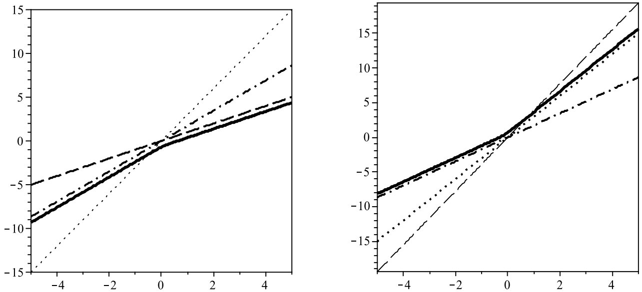
There are two exact de Sitter solutions:

I. $\alpha(t) = H_\Lambda t, \quad \phi(t) = \phi_0 = const,$

II. $\alpha(t) = \dfrac{t}{\sqrt{3|\eta|}}, \quad \phi(t) = M_{\rm Pl}\left|\dfrac{3\eta H_\Lambda^2 - 1}{\eta}\right|^{1/2} t,$

$$H_\Lambda = \sqrt{\Lambda/3}$$

Plots of $\alpha(t)$ in case $\eta > 0$, $\epsilon = 1$, $V = M_{\rm Pl}^2\Lambda$

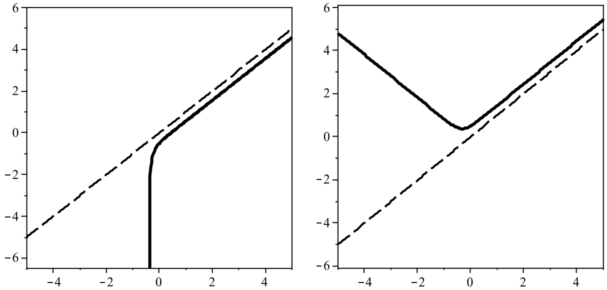


(a) $H_\Lambda^2 < \dot\alpha^2 < 1/9\eta$ (b) $1/9\eta < \dot\alpha^2 < 1/3\eta < H_\Lambda^2$

De Sitter asymptotics:
$\alpha_1(t) = H_\Lambda t$ *(dashed),*
$\alpha_2(t) = t/\sqrt{9\eta}$ *(dash-dotted),*
$\alpha_3(t) = t/\sqrt{3\eta}$ *(dotted).*

Cosmological models: IV. Cosmological constant

Plots of $\alpha(t)$ in cases $\eta > 0$, $\epsilon = -1$ and $\eta < 0$, $\epsilon = 1$



(a) $\eta < 0$, $\epsilon = 1$ (b) $\eta > 0$, $\epsilon = -1$

De Sitter asymptotic:
$\alpha_1(t) = H_\Lambda t$ *(dashed).*

Role of potential

$$S = \int d^4x\sqrt{-g}\left\{M_{\rm Pl}^2 R - [g^{\mu\nu} + \eta G^{\mu\nu}]\phi_{,\mu}\phi_{,\nu} - 2V(\phi)\right\}$$

$\nearrow$

What a role does a potential play in cosmological models with the nonminimal kinetic coupling?

Models with the quadratic potential $V(\phi) = \frac{1}{2}m^2\phi^2$

Primary (early-time) "kinetic" inflation:

$$H_{t\to-\infty} \approx \frac{1}{\sqrt{9\eta}}(1 + \tfrac{1}{2}\eta m^2)$$

Late-time cosmological scenarios:

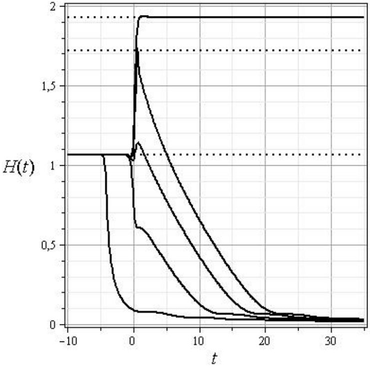
Oscillatory asymptotic or "graceful" exit from inflation

$$H_{t\to\infty} \approx \frac{2}{3t}\left[1 - \frac{\sin 2mt}{2mt}\right]$$

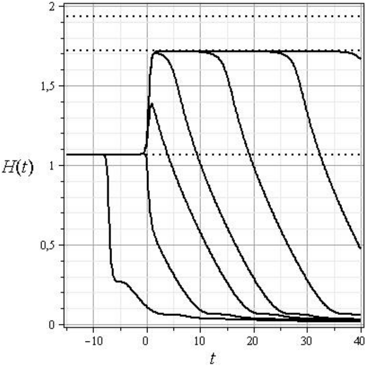
quasi-de Sitter asymptotic or secondary inflation

$$H_{t\to\infty} \approx \frac{1}{\sqrt{3\eta}}\left(1 \pm \sqrt{\tfrac{1}{6}\eta m^2}\right)$$

Cosmological models: Power-law potential



Initial conditions
$\phi_0 = \dot{\phi}_0$



Initial conditions
$\phi_0 = -\dot{\phi}_0$

De Sitter asymptotics: $H_{t\to-\infty} \approx 1/\sqrt{9\eta}(1 + \frac{1}{2}\eta m^2)$,

$$H_{t\to\infty} \approx 1/\sqrt{3\eta}\left(1 \pm \sqrt{\tfrac{1}{6}\eta m^2}\right).$$

The FLRW ansatz for the metric:

$$ds^2 = -dt^2 + \mathrm{a}^2(t)\left[\frac{dr^2}{1-Kr^2} + r^2(d\vartheta^2 + \sin^2\vartheta d\varphi^2)\right],$$

$\mathrm{a}(t)$ *cosmological factor*, $H = \dot{\mathrm{a}}/\mathrm{a}$ *Hubble parameter*

Gravitational equations:

$$-3M_{\mathrm{Pl}}^2\left(H^2 + \frac{K}{\mathrm{a}^2}\right) + \frac{1}{2}\,\varepsilon\,\psi^2 - \frac{3}{2}\,\eta\,\psi^2\left(3H^2 + \frac{K}{\mathrm{a}^2}\right) + \Lambda + \rho = 0,$$

$$-M_{\mathrm{Pl}}^2\left(2\dot{H} + 3H^2 + \frac{K}{\mathrm{a}^2}\right) - \frac{1}{2}\,\varepsilon\,\psi^2 - \eta\,\psi^2\left(\dot{H} + \frac{3}{2}\,H^2 - \frac{K}{\mathrm{a}^2} + 2H\frac{\dot{\psi}}{\psi}\right) + \Lambda - p = 0.$$

The scalar field equation:

$$\frac{1}{\mathrm{a}^3}\frac{d}{dt}\left(\mathrm{a}^3\left(3\eta\left(H^2 + \frac{K}{\mathrm{a}^2}\right) - \varepsilon\right)\psi\right) = 0,$$

where $\psi = \dot{\phi}$, and $\phi = \phi(t)$ is a homogeneous scalar field

Screening properties of Horndeski model

The first integral of the scalar field equation:

$$\mathrm{a}^3\left(3\eta\left(H^2+\frac{K}{\mathrm{a}^2}\right)-\varepsilon\right)\psi=Q,$$

where Q is the Noether charge associated with the shift symmetry $\phi\to\phi+\phi_0$.

Let $Q=0$. One finds in this case two different solutions:

GR branch: $\psi=0 \quad\Longrightarrow\quad H^2+\frac{K}{\mathrm{a}^2}=\frac{\Lambda+\rho}{3M_{\mathrm{Pl}}^2}$

Screening branch: $H^2+\frac{K}{\mathrm{a}^2}=\frac{\varepsilon}{3\eta} \quad\Longrightarrow\quad \psi^2=\frac{\eta\,(\Lambda+\rho)-\varepsilon\,M_{\mathrm{Pl}}^2}{\eta\,(\varepsilon-3\eta\,K/\mathrm{a}^2)}$

NOTICE: The role of the cosmological constant in the screening solution is played by $\varepsilon/3\eta$ while the Λ-term is screened and makes no contribution to the universe acceleration.

Note also that the matter density ρ is screened in the same sense.

Screening properties of Horndeski model

Let $Q \neq 0$, then

$$\psi = \frac{Q}{\mathrm{a}^3\,\left[3\eta\left(H^2 + \frac{K}{\mathrm{a}^2}\right) - \varepsilon\right]},$$

and the modified Friedmann equation reads

$$3M_{\mathrm{Pl}}^2\left(H^2 + \frac{K}{\mathrm{a}^2}\right) = \frac{Q^2\left[\varepsilon - 3\eta\left(3H^2 + \frac{K}{\mathrm{a}^2}\right)\right]}{2a^6\,\left[\varepsilon - 3\eta\left(H^2 + \frac{K}{\mathrm{a}^2}\right)\right]^2} + \Lambda + \rho.$$

Introducing dimensionless values and density parameters

$$H^2 = H_0^2\,y,\;\; \mathrm{a} = \mathrm{a}_0\,a\,,\;\; \rho_{\mathrm{cr}} = 3M_{\mathrm{Pl}}^2 H_0^2\,,\;\; \eta = \frac{\varepsilon}{3\eta\,H_0^2}\,,$$

$$\Omega_0 = \frac{\Lambda}{\rho_{\mathrm{cr}}},\; \Omega_2 = -\frac{K}{H_0^2\mathrm{a}_0^2},\; \Omega_6 = \frac{Q^2}{6\eta\,\mathrm{a}_0^6\,H_0^2\,\rho_{\mathrm{cr}}},\; \rho = \rho_{\mathrm{cr}}\left(\frac{\Omega_4}{a^4} + \frac{\Omega_3}{a^3}\right)$$

gives

the master equation:

$$y = \Omega_0 + \frac{\Omega_2}{a^2} + \frac{\Omega_3}{a^3} + \frac{\Omega_4}{a^4} + \frac{\Omega_6\left[\eta - 3y + \frac{\Omega_2}{a^2}\right]}{a^6\left[\eta - y + \frac{\Omega_2}{a^2}\right]^2}$$

Asymptotical behavior: Late time limit $a \to \infty$

GR branch:

$$y = \Omega_0 + \frac{\Omega_2}{a^2} + \frac{\Omega_3}{a^3} + \frac{\Omega_4}{a^4} + \frac{(\eta - 3\,\Omega_0)\,\Omega_6}{(\,\Omega_0 - \eta)^2\,a^6} + \mathcal{O}\left(\frac{1}{a^7}\right) \Longrightarrow \quad H^2 \to \Lambda/3$$

Notice: The GR solution is stable (no ghost) if and only if $\eta > \Omega_0$.

Screening branches:

$$y_\pm = \eta + \frac{\Omega_2}{a^2} \pm \frac{\chi}{(\,\Omega_0 - \eta)\,a^3} \pm \frac{\Omega_2\Omega_6}{\chi\,a^5} - \frac{\Omega_6(\eta - 3\Omega_0) \pm \Omega_3\chi}{2(\Omega_0 - \eta)^2\,a^6} + \mathcal{O}\left(\frac{1}{a^7}\right)$$

$$\Longrightarrow \quad H^2 \to \varepsilon/3\alpha$$

Notice: The screening solutions are stable (no ghost) if and only if $0 < \eta < \Omega_0$.

Asymptotical behavior: The limit $a \to 0$

GR branch:

$$y = \frac{\Omega_4}{a^4} + \frac{\Omega_3}{a^3} + \frac{\Omega_2\Omega_4 - 3\Omega_6}{\Omega_4 a^2} + \frac{3\Omega_3\Omega_6}{\Omega_4 a} + \mathcal{O}(1)$$

Notice: The GR solution is unstable

Screening branch:

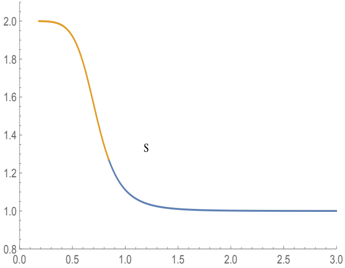
$$y_+ = \frac{3\Omega_6}{\Omega_4\, a^2} - \frac{3\Omega_3\Omega_6}{\Omega_4^2\, a} + \frac{5}{3}\,\eta + \frac{3\Omega_6\Omega_3^2 + 9\Omega_6^2}{\Omega_4^3} + \mathcal{O}(a),$$

$$y_- = \frac{1}{\sqrt{9\eta}} + \frac{4\,\eta^2}{27\ \Omega_6}\left(\Omega_4\, a^2 + \Omega_3\, a^3\right) + \mathcal{O}(a^4)$$

Notice: Both screening solutions are stable

Global behavior

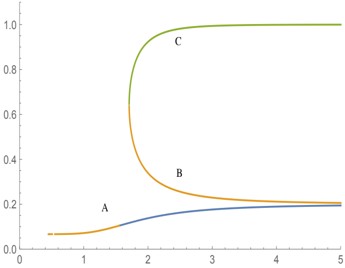
$$y = \Omega_0 + \frac{\Omega_2}{a^2} + \frac{\Omega_3}{a^3} + \frac{\Omega_4}{a^4} + \frac{\Omega_6\left[\eta - 3y + \frac{\Omega_2}{a^2}\right]}{a^6\left[\eta - y + \frac{\Omega_2}{a^2}\right]^2}$$



Solutions $y(a)$ for $\Omega_0 = \Omega_6 = 1$, $\Omega_2 = 0$, $\Omega_3 = \Omega_4 = 0$ and for $\eta = 6$

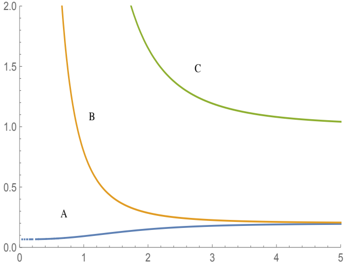
Global behavior

$$y = \Omega_0 + \frac{\Omega_2}{a^2} + \frac{\Omega_3}{a^3} + \frac{\Omega_4}{a^4} + \frac{\Omega_6\left[\eta - 3y + \frac{\Omega_2}{a^2}\right]}{a^6\left[\eta - y + \frac{\Omega_2}{a^2}\right]^2}$$



Solutions $y(a)$ for $\Omega_0 = \Omega_6 = 1$, $\Omega_2 = 0$, $\Omega_3 = \Omega_4 = 0$, $\eta = 0.2$

$$y = \Omega_0 + \frac{\Omega_2}{a^2} + \frac{\Omega_3}{a^3} + \frac{\Omega_4}{a^4} + \frac{\Omega_6\left[\eta - 3y + \frac{\Omega_2}{a^2}\right]}{a^6\left[\eta - y + \frac{\Omega_2}{a^2}\right]^2}$$



Solutions $y(a)$ for $\Omega_0 = \Omega_6 = 1$, $\Omega_3 = 5$, $\Omega_4 = 0$, $\eta = 0.2$. One has $\Omega_2 = 0$.

Intermediate Summary

- The nonminimal kinetic coupling provides an *essentially new* inflationary mechanism which does not need any fine-tuned potential.
- At early cosmological times the coupling η-terms in the field equations are dominating and provide the quasi-De Sitter behavior of the scale factor: $a(t) \propto e^{H_\eta t}$ with $H_\eta = 1/\sqrt{9\eta}$.
- The model provides a natural mechanism of epoch change without any fine-tuned potential.
- The model with nonminimal kinetic coupling is distinguished by the *screening property*. This property suggests that Λ is actually irrelevant and hence there is no need to explain its value.

Scalar perturbations

Scalar perturbations in the Newtonian gauge:

$$ds^2 = -(1+2\Psi)dt^2 + a^2(t)(1+2\Phi)\delta_{ij}dx^i dx^j,$$
$$\phi = \phi_0 + \delta\phi = \phi_0(1+\varphi),$$
$$\Psi(t,\mathbf{x}) \ll 1,\ \Phi(t,\mathbf{x}) \ll 1,\ \varphi(t,\mathbf{x}) \ll 1$$

Fourier transformations: $\Psi(t,\mathbf{x}) = \int d\mathbf{k}e^{i\mathbf{kx}}\Psi(t,\mathbf{k})$ and so on

Scalar modes:

$$-3H(\dot{\Psi} - H\Phi) - \frac{k^2}{a^2}\Psi = 4\pi\Big[\dot{\phi}^2\Phi - \dot{\phi}\delta\dot{\phi}$$
$$+\eta\left(9H\dot{\phi}^2\dot{\Psi} - 18H^2\dot{\phi}^2\Phi + \frac{k^2}{a^2}\dot{\phi}^2\Psi + 9H^2\dot{\phi}\delta\dot{\phi} + 2\frac{k^2}{a^2}H\dot{\phi}\delta\phi\right)\Big],$$
$$\dot{\Psi} - H\Phi = 4\pi\left[-\dot{\phi}\delta\phi + \eta\left(3H\dot{\phi}^2\Phi - \dot{\phi}^2\dot{\Psi} - 2H\dot{\phi}\delta\dot{\phi} + 3H^2\dot{\phi}\delta\phi\right)\right],$$
$$\Phi + \Psi = -4\pi\eta\left[\dot{\phi}^2(\Phi - \Psi) + 2(\ddot{\phi} + H\dot{\phi})\delta\phi\right]$$

Scalar perturbations

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$$\Psi(t,\mathbf{x}) \ll 1,\ \ \Phi(t,\mathbf{x}) \ll 1,\ \ \varphi(t,\mathbf{x}) \ll 1$$

Fourier transformations: $\Psi(t,\mathbf{x}) = \int d\mathbf{k} e^{i\mathbf{k}\mathbf{x}}\Psi(t,\mathbf{k})$ and so on

Scalar modes:

$$-3H(\dot\Psi - H\Phi) - \frac{k^2}{a^2}\Psi = 4\pi\Big[\dot\phi^2\Phi - \dot\phi\delta\dot\phi$$
$$+\eta\left(9H\dot\phi^2\dot\Psi - 18H^2\dot\phi^2\Phi + \frac{k^2}{a^2}\dot\phi^2\Psi + 9H^2\dot\phi\delta\dot\phi + 2\frac{k^2}{a^2}H\dot\phi\delta\phi\right)\Big],$$
$$\dot\Psi - H\Phi = 4\pi\left[-\dot\phi\delta\phi + \eta\left(3H\dot\phi^2\Phi - \dot\phi^2\dot\Psi - 2H\dot\phi\delta\dot\phi + 3H^2\dot\phi\delta\phi\right)\right],$$
$$\Phi + \Psi = -4\pi\eta\left[\dot\phi^2(\Phi-\Psi) + 2(\ddot\phi + H\dot\phi)\delta\phi\right]$$

Notice: $\Psi = -\Phi$ if $\eta = 0$, but generally $\Psi \neq -\Phi$!

Scalar perturbations in the inflationary epoch

On the inflationary stage the unperturbed solutions are

$$a(t) = a_i e^{H_\eta (t-t_i)}, \quad \phi(t) = \phi_i e^{-3H_\eta (t-t_i)}, \quad \text{where} \quad H_\eta = \frac{1}{\sqrt{9\eta}}.$$

t_i – beginning of inflation

Scalar perturbations on the inflationary stage

$$3H_\eta(\dot\Psi - H_\eta\Phi) + \frac{k^2}{a^2}\Psi = -4\pi\phi^2\left[9H_\eta(\dot\Psi - H_\eta\Phi) + \frac{k^2}{a^2}\chi\right],$$
$$\dot\Psi - H_\eta\Phi = 4\pi\phi^2\left[3H_\eta\Phi - \dot\chi\right],$$
$$\Phi + \Psi = -4\pi\phi^2\left[\Phi + \Psi - 2\chi\right].$$

where $\chi = \Psi - \frac{2}{3}\varphi$ is a combination of pertubations

Scalar perturbations in the inflationary epoch

Resolving with the respect of Ψ and Φ we find

Scalar perturbations of metric:

$$\frac{d\Psi}{d\tau}=\Phi-\frac{k^2\,e^{-2\tau}}{6a_i^2H_\eta^2}\,\frac{\Phi+3\Psi+4\pi\phi_i^2e^{-6\tau}(\Phi+\Psi)}{1+12\pi\phi_i^2e^{-6\tau}},$$

$$\frac{d\Phi}{d\tau}=-\frac{7\Phi+6\Psi-20\pi\phi_i^2e^{-6\tau}\,\Phi}{1+4\pi\phi_i^2e^{-6\tau}} +\frac{k^2\,e^{-2\tau}}{6a_i^2H_\eta^2}\,\frac{3(\Phi+3\Psi)+8\pi\phi_i^2e^{-6\tau}(2\Phi+3\Psi)+16\pi^2\phi_i^4e^{-12\tau}(\Phi+\Psi)}{(1+4\pi\phi_i^2e^{-6\tau})(1+12\pi\phi_i^2e^{-6\tau})},$$

where we denote $\tau=H_\eta(t-t_i)$

Scalar perturbations in the inflationary epoch

Assume that $8\pi\phi_i^2 \equiv \frac{\phi_i^2}{M_{Pl}^2} \ll 1$, then

Metric perturbations

$$\frac{d\Psi}{d\tau} = \Phi - \frac{k^2\, e^{-2\tau}}{6a_i^2 H_\eta^2}\,(\Phi + 3\Psi),$$

$$\frac{d\Phi}{d\tau} = -(7\Phi + 6\Psi) + \frac{k^2\, e^{-2\tau}}{2a_i^2 H_\eta^2}\,(\Phi + 3\Psi).$$

Limiting case:

A. $k/a_i \ll H_\eta$ (long-wave modes outside the Hubble horizon)

$$\frac{d\Psi}{d\tau}=\Phi-\frac{k^2\,e^{-2\tau}}{6a_i^2H_\eta^2}\,(\Phi+3\Psi),$$
$$\frac{d\Phi}{d\tau}=-(7\Phi+6\Psi)+\frac{k^2\,e^{-2\tau}}{2a_i^2H_\eta^2}\,(\Phi+3\Psi).$$

$$\Psi=\tfrac{1}{5}(6\Psi_i+\Phi_i)e^{-H_\eta(t-t_i)}-\tfrac{1}{5}(\Psi_i+\Phi_i)e^{-6H_\eta(t-t_i)},$$
$$\Phi=-\tfrac{1}{5}(6\Psi_i+\Phi_i)e^{-H_\eta(t-t_i)}+\tfrac{6}{5}(\Psi_i+\Phi_i)e^{-6H_\eta(t-t_i)},$$

$\Psi_i=\Psi(t_i)\ll 1,\ \Phi_i=\Phi(t_i)\ll 1,\quad t=t_i$ – beginning of inflation

Perturbs in course of inflation $t>t_i$: $\Psi=-\Phi\sim e^{-H_\eta(t-t_i)}\sim a^{-1}$

NOTICE: Scalar modes $k/a_i \ll H_\eta$ are exponentially decaying!

Scalar perturbations: Modes inside the Hubble horizon

B. $k/a_i \gg H_\eta$ (short-wave modes inside the Hubble horizon)

$$\frac{d\Psi}{d\tau} = \Phi - \frac{k^2\, e^{-2\tau}}{6a_i^2 H_\eta^2}\,(\Phi + 3\Psi),$$

$$\frac{d\Phi}{d\tau} = -(7\Phi + 6\Psi) + \frac{k^2\, e^{-2\tau}}{2a_i^2 H_\eta^2}\,(\Phi + 3\Psi).$$

$$\Psi = C_1 + C_2 e^{-2\tau}, \quad \Phi = -3\Big[C_1 + C_2 e^{-2\tau}\Big] + \frac{12 a_i^2 H_\eta^2}{k^2} C_2,$$

where $C_1 = \Psi_i - C_2, \quad C_2 = \frac{k^2}{12 a_i^2 H_\eta^2}\big(3\Psi_i + \Phi_i\big)$

During the inflation, $t > t_i$, metric perturbations tend to

$$\Psi_f \sim \Phi_f \sim \frac{k^2}{a_i^2 H_\eta^2}\big(3\Psi_i + \Phi_i\big)$$

NOTICE: During the inflation the initial short-wave perturbations are amplifying by the factor $\frac{k^2}{a_i^2 H_\eta^2}$!

Estimations:

$$\Psi_{final} \sim \Phi_{final} \sim 1 \quad \Longrightarrow \quad \frac{k^2}{a_i^2 H_\eta^2}\left(3\Psi_i + \Phi_i\right) \sim 1$$

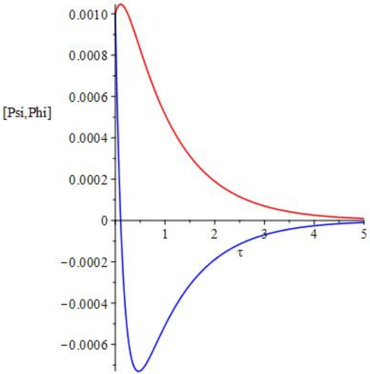
$$\Psi_i \sim \Phi_i \sim [Planck] \quad \Longrightarrow \quad \frac{k^2}{a_i^2 H_\eta^2} \sim [Planck]^{-1} \quad \Longrightarrow \quad \lambda_{mode} \sim L_{Pl}$$

NOTICE: Only modes with very short (Planckian) initial wavelengths are able to amplify enough during the kinetic inflation.

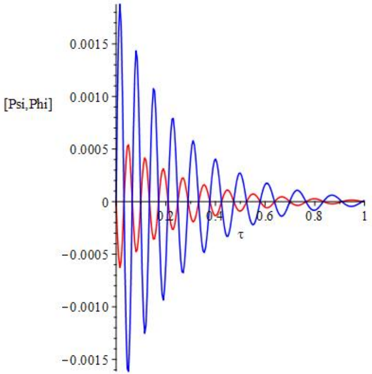
TENDENCY: During the inflation, modes with short wavelength are stretching and come outside the Hubble horizon. After they have gone outside the Hubble horizon, they are exponential decaying.

Scalar perturbations: Numerical analysis

Examples of numerical analysis for scalar mode evolution:



Longwave modes



Shortwave modes

Tensor perturbations

Perturbed metric:

$$ds^2 = -dt^2 + a^2(t)\big(\delta_{ij} + h_{ij}\big)dx^i dx^j$$

Small transversal traceless pertubarions:

$$h_{ij}(t,\mathbf{x}) \ll 1, \quad \partial_i h_{ij} = 0, \quad h_{ii} = 0$$

Two polarizations: $$h_{ij} = \sum_{A=+,\times} e_{ij}^{(A)} h^{(A)}$$

Fourier transformation: $$h^{(A)}(t,\mathbf{x}) = \int d\mathbf{k} e^{i\mathbf{kx}} h^{(A)}(t,\mathbf{k})$$

Equation for tensor modes $h^{(A)}(t,\mathbf{k})$:

$$(1+4\pi\eta\dot{\phi}^2)\ddot{h} + \left(3H(1+4\pi\eta\dot{\phi}^2) + 8\pi\eta\dot{\phi}\ddot{\phi}\right)\dot{h} + \frac{k^2}{a^2}(1-4\pi\eta\dot{\phi}^2)h = 0$$

Tensor perturbations on the inflationary stage

Unperturbed solutions on the inflationary stage:

$$a(t) = a_i e^{H_\eta (t-t_i)}, \quad \phi(t) = \phi_i e^{-3H_\eta (t-t_i)}, \quad H_\eta = \tfrac{1}{\sqrt{9\eta}}$$

Smallness of derivatives of the scalar field: $8\pi\eta\dot{\phi}^2 \ll 1$

Equation for tensor modes on the inflationary stage:

$$\frac{d^2h}{d\tau^2} + 3\frac{dh}{d\tau} + \frac{k^2}{a_i^2 H_\eta^2} e^{-2\tau} h = 0.$$

$\tau = H_\eta (t - t_i)$

NOTICE: Compare with GR: $\ddot{h} + 3H\dot{h} + \frac{k^2}{a^2} h = 0$

Tensor perturbations: Modes outside and inside the Hubble horizon

A. $k/a_i \ll H_\eta$ (long-wave modes outside the Hubble horizon)

$$\frac{d^2h}{d\tau^2} + 3\frac{dh}{d\tau} + \frac{k^2}{a_i^2 H_\eta^2} e^{-2\tau} h = 0$$

$$h = C_1 + C_2 e^{-3\tau}$$

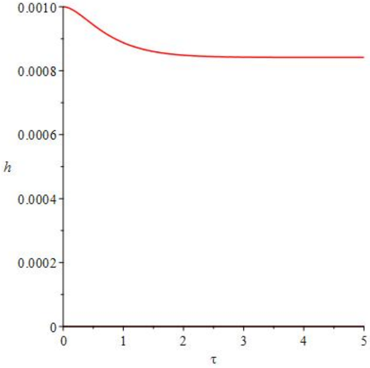
NOTICE: During the inflation longwave tensor modes tend to constant values (constant modes)

B. $k/a_i \gg H_\eta$ (short-wave modes inside the Hubble horizon)

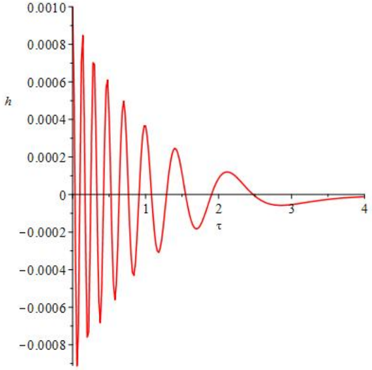
$$\frac{d^2h}{d\tau^2} + 3\frac{dh}{d\tau} + \frac{k^2}{a_i^2 H_\eta^2} e^{-2\tau} h = 0$$

NOTICE: Shortwave tensor modes are oscillating and damping

Examples of numerical analysis for tensor mode evolution:



Longwave modes



Shortwave modes

Final conclusions

- The behavior of *tensor* modes during the kinetic inflation is analogous to that in "usual" slow-roll inflation.
- Long-wave scalar modes with $k/a_i \ll H_\eta$ are exponentially decaying during the kinetic inflation. Therefore, the large-scale structure of the Universe keeps to be homogeneous and isotropic.
- During the kinetic inflation the initial short-wave scalar modes are amplifying by the factor $\frac{k^2}{a_i^2 H_\eta^2}$. Then, they stretch, come outside the Hubble horizon and exponential decay. Only modes with very short (Planckian) initial wavelengths are able to amplify enough during the kinetic inflation.

THANKS FOR YOUR ATTENTION!